Indefinite Integration

Question1

If $\frac{x^4}{(x-1)(x-2)}=f(x)+\frac{A}{x-1}+\frac{B}{x-2}$**, then** $f(-2)+A+B=$

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Options:

A.

32

B.

28

C.

22

D.

20

Answer: D

Solution:

$\frac{x^4}{(x-1)(x-2)}=f(x)+\frac{A}{(x-1)}+\frac{B}{(x-2)}$

$\Rightarrow x^4=f(x)(x-2)(x-1)+A(x-2)+B(x-1)$

at $x=1$

$1=-A\Rightarrow A=-1$

at $x=2$

$16=B(2-1)\Rightarrow B=16$

Put the values of A and B and put

$$\begin{aligned} & x=-2 \\ & 16=f(-2)(-3)(-4)-(-4)+16(-3) \\ \Rightarrow\quad & 12f(-2)=60 \\ & \Rightarrow f(-2)=5 \end{aligned}$$

Hence, $f(-2)+A+B=5-1+16=20$

Question2

If $\int \frac{x^4+1}{x^2+1} dx = Ax^3 + Bx^2 + Cx + D\tan^{-1}x + E$**, then** $A + B + C + D =$

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Options:

A.

$\frac{3}{2}$

B.

$\frac{4}{3}$

C.

$\frac{1}{3}$

D.

$\frac{2}{3}$

Answer: B

Solution:

$$\begin{aligned} \frac{x^4+1}{x^2+1} &= \frac{x^4-1+2}{x^2+1} \\ &= \frac{(x^4-1)}{x^2+1} + \frac{2}{x^2+1} \\ &= \frac{(x^2-1)(x^2+1)}{x^2+1} + \frac{2}{x^2+1} \\ &= x^2 - 1 + \frac{2}{x^2+1} \end{aligned}$$

So, $\int \frac{x^4+1}{x^2+1} dx = \int \left(x^2 - 1 + \frac{2}{x^2+1}\right) dx$

$= \frac{x^3}{3} - x + 2\tan^{-1}x + c$

Comparing with

$Ax^3 + Bx^2 + Cx + D\tan^{-1}x + E$

We get, $A = 1/3, B = 0, C = -1, D = 2$ Therefore,

$A + B + C + D = \frac{1}{3} + 0 - 1 + 2 = 4/3$

Question3

If $\int \frac{x^2-x+2}{x^2+x+2} dx = x - \log(f(x)) + \frac{2}{\sqrt{7}} \tan^{-1}(g(x)) + c,$ then $f(-1) + \sqrt{7} g(-1) =$

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Options:

A.

1

B.

0

C.

-1

D.

2

Answer: A

Solution:

$$\begin{aligned} \frac{x^2-x+2}{x^2+x+2} &= 1 - \frac{2x}{x^2+x+2} \\ &= 1 - \frac{2x+1-1}{x^2+x+2} \\ &= 1 - \frac{2x+1}{x^2+x+2} - \frac{1}{x^2+x+2} \\ &= 1 - \frac{2x+1}{x^2+x+2} - \frac{1}{\left(x+\frac{1}{2}\right)^2 + \frac{7}{4}} \end{aligned}$$

$$\begin{aligned} & \therefore \int \frac{x^2-x+2}{x^2+x+2} dx \\ & = \int \left(1 - \frac{2x+1}{x^2+x+2} - \frac{1}{\left(x+\frac{1}{2}\right)^2 + \frac{7}{4}}\right) dx \\ & = \int 1 \cdot dx - \int \frac{2x+1}{x^2+x+2} dx - \int \frac{1}{\left(x+\frac{1}{2}\right)^2 + \frac{7}{4}} \cdot dx \end{aligned}$$

let $x^2+x+2 = u \Rightarrow (2x+1) dx = du$

$$\begin{aligned} & = x - \int \frac{1}{u} \cdot du - \frac{1}{\sqrt{7/4}} \tan^{-1}\left(\frac{x+1/2}{\sqrt{7/4}}\right) \\ & = x - \ln\left(x^2+x+2\right) - \frac{2}{\sqrt{7}} \tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right) \end{aligned}$$

Comparing with

$x-\log(f(x))+\frac{2}{\sqrt{7}}\tan^{-1}(g(x))$

$f(x)=x^2+x+2$ and $g(x)=\frac{2x+1}{\sqrt{7}}$

then, $f(-1)=(-1)^2-1+2=2$

and $g(-1)=\frac{2(-1)+1}{\sqrt{7}}=-1/\sqrt{7}$

Hence, $f(-1)+\sqrt{7}\cdot g(-1)$

$=2+\sqrt{7}(-1/\sqrt{7})=1$

Question4

$\int \sec\left(x-\frac{\pi}{3}\right)\sec\left(x+\frac{\pi}{6}\right)dx=$

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Options:

A.

$\log\left|\frac{\sec\left(x-\frac{\pi}{3}\right)}{\sec\left[x+\frac{\pi}{6}\right]}\right|+C$

B.

$\log\left|\frac{\cos\left(x-\frac{\pi}{3}\right)}{\cos\left(x+\frac{\pi}{6}\right)}\right|+C$

C.

$\log\left|\frac{\operatorname{cosec}\left(x-\frac{\pi}{3}\right)}{\operatorname{cosec}\left(x+\frac{\pi}{6}\right)}\right|+C$

D.

$\log\left|\frac{\sin\left(x-\frac{\pi}{3}\right)}{\sin\left(x+\frac{\pi}{6}\right)}\right|+C$

Answer: B

Solution:

In $\sec\left(x-\frac{\pi}{3}\right)\sec\left(x+\frac{\pi}{6}\right)$

Let $A=x-\frac{\pi}{3}$ and $B=x+\frac{\pi}{6}$

then $B-A=x/2\Rightarrow B=A+\pi/2$

So, $\sec B=\sec\left(\frac{\pi}{2}+A\right)=-\operatorname{cosec} A$

Then, $\sec A\cdot\sec B=\sec A(-\operatorname{cosec} A)$

$=-\sec A\cdot\operatorname{cosec} A$

Now, $-\sec A \cdot \operatorname{cosec} A=\frac{-1}{\sin A \cdot \cos A}$

$$=\frac{-2}{\sin 2A}=-2 \operatorname{cosec} 2A$$

$$\therefore \int \sec\left(x-\frac{\pi}{3}\right) \sec\left(x+\frac{\pi}{6}\right) dx$$

$$=-2 \int \operatorname{cosec}\left(2x-\frac{2\pi}{3}\right) dx$$

Let $2x-\frac{2\pi}{3}=u$

$$\Rightarrow 2 \cdot dx=du=-\int \operatorname{cosec} u \cdot du$$

$$=-\ln|\operatorname{cosec}(u)-\cot(u)|+c$$

and $\operatorname{cosec}(u)-\cot(u)=\tan(u/2)$

$$=-\ln\left|\tan\left(\frac{u}{2}\right)\right|+C=-\ln\left|\tan\left(x-\frac{\pi}{3}\right)\right|+c$$

$$=\ln\left|\cot\left(x-\frac{\pi}{3}\right)\right|+C=\ln\left|\frac{\cos\left(x-\frac{\pi}{3}\right)}{\sin(x-\pi/3)}\right|+c$$

$$\because \sin\left(x-\frac{\pi}{3}\right)=\sin\left(\frac{\pi}{2}+\left(x-\frac{\pi}{6}\right)\right)$$

$$=\cos\left(x-\frac{\pi}{6}\right)$$

$$=\ln\left|\frac{\cos(x-\pi/3)}{\cos(x-\pi/6)}\right|+c$$

Question5

If $\int \frac{a \cos x+3 \sin x}{5 \cos x+2 \sin x} dx=\frac{26}{29} x-\frac{k}{29} \log|5 \cos x+2 \sin x|+\ldots$ **then** $|a+k|=$

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Options:

A.

3

B.

11

C.

12

D.

2

Answer: B

Solution:

$$\int \frac{a\cos x + 3\sin x}{5\cos x + 2\sin x} \cdot dx$$
$$= \frac{26}{29}x - \frac{k}{29}\log|5\cos x + 2\sin x| + C$$
$$\because \frac{d}{dx}(5\cos x + 2\sin x) = -5\sin x + 2\cos x$$
$$\text{let } a\cos x + 3\sin x = A(-5\sin x + 2\cos x) + B(5\cos x + 2\sin x)$$

By comparing

$-5A + 2B = 3 \quad \ldots(i)$

$2A + 5B = a \quad \ldots(ii)$

[Eq. (ii)] $\times$ 2 - [Eq. (i)] $\times$ 5

$29A = 2a - 15 \Rightarrow A = \frac{2a-15}{29}$

So, $I = A\log|5\cos x + 2\sin x| + Bx + C$

Comparing with the original form

We get $A = -\frac{k}{29}, B = \frac{26}{29}$

So, $\quad A = \frac{2a-15}{29} = \frac{-k}{29}$

$\Rightarrow k = 15 - 2a \Rightarrow |a + k| = |15 - a|$

From Eq. (ii), we get $2\left(\frac{2a-15}{29}\right) + 3\left(\frac{26}{29}\right) = a$

$\Rightarrow 4a - 30 + 130 = 29a \Rightarrow a = 4$

Hence, $|a + k| = |15 - a| = 11$

Question6

If $\int \frac{dx}{1-\sin^4 x} = A\tan x + B\tan^{-1}(\sqrt{2}\tan x) + C$**, then** $A^2 - B^2 =$

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Options:

A.

$\frac{1}{2}$

B.

$\frac{3}{4}$

C.

$\frac{1}{4}$

D.

$\frac{1}{8}$

Answer: D

Solution:

$$\because 1-\sin^4 x=\left(1-\sin^2 x\right)\left(1+\sin^2 x\right)$$
$$=\cos^2 x\left(1+\sin^2 x\right)$$
$$\therefore \int \frac{1}{1-\sin^4 x} dx=\int \frac{1}{\cos^2 x \cdot\left(1+\sin^2 x\right)} \cdot dx$$
$$=\int \frac{\sec^2 x}{1+\sin^2 x} dx$$

Let, $\tan x=t$

$\Rightarrow \frac{1}{1+\sin^2 x}=\frac{1+t^2}{1+2t^2} \quad \left\{\because \sin x=\frac{1}{\sqrt{1+r^2}}\right\}$

and $\sec^2 x \cdot dx=dt$

$$=\int \frac{1+t^2}{1+2t^2} \cdot dt$$
$$\because \frac{1+t^2}{1+2t^2}=\frac{1}{2}\left(1+\frac{1}{1+2t^2}\right)$$
$$\therefore \quad I=\frac{1}{2} \int\left(1+\frac{1}{1+2t^2}\right) dt$$
$$=\frac{1}{2}\left(t+\frac{1}{\sqrt{2}} \tan^{-1}(\sqrt{2} t)\right)+C$$
$$=\frac{1}{2} \tan x+\frac{1}{2\sqrt{2}} \tan^{-1}(\sqrt{2} \tan x)+C$$
$$A \tan x+B \tan^{-1}(\sqrt{2} \tan x)+C$$

We get, $A=\frac{1}{2}, B=\frac{1}{2\sqrt{2}}$

$$\therefore A^2-B^2=\frac{1}{4}-\frac{1}{8}=\frac{1}{8}$$

Question7

If $\frac{2x^4-3x^2+4}{(x^2+1)(x^2+2)}=a+\frac{px+q}{x^2+1}+\frac{mx+n}{x^2+2}$, then $\frac{n}{q}=$

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Options:

A.

$p+m-a$

B.

$\frac{p+m}{a}$

C.

$\frac{a}{p+m}$

D.

$p+m+a$

Answer: A

Solution:

Given,

$$\begin{aligned}
&\frac{2x^4-3x^2+4}{(x^2+1)(x^2+2)}=\frac{2x^4-3x^2+4}{x^4+3x^2+2}\\
&=\frac{2x^4+6x^2+4}{x^4+3x^2+2}-\frac{9x^2}{x^4+3x^2+2}\\
&=2-\frac{9x^2}{x^4+3x^2+2}\\
\therefore\ &\frac{9x^2}{x^4+3x^2+2}=\frac{A}{x^2+1}+\frac{B}{x^2+2}\\
\Rightarrow\ & 9x^2=A(x^2+2)+B(x^2+1)\\
\Rightarrow\ & 9=A+B \text{ and } 2A+B=0\\
\Rightarrow\ & B=-2A\\
\therefore\ & A=-9, B=18\\
\therefore\ &\frac{2x^4-3x^2+4}{(x^2+1)(x^2+2)}=2+\frac{9}{x^2+1}-\frac{18}{x^2+2}\\
& a=2, p=0, q=9, m=0, n=-18\\
\therefore\ & n/q=-2
\end{aligned}$$

$p+m-a=0+0-2=-2$

Question8

$\int(\log 2x)^3 dx=$

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Options:

A.

$x\left((\log 2x)^3-3(\log 2x)^2+6(\log 2x)-6\right]+C$

B.

$\frac{x}{4}\left[4(\log 2x)^3-6(\log 2x)^2+6(\log 2x)-3\right]+C$

C.

$\frac{x}{2}\left[(\log 2x)^3-3(\log 2x)^2+3(\log 2x)-6\right]+C$

D.

$x\left[(\log 2x)^3-6(\log 2x)^2+18(\log 2x)-54\right]+C$

Answer: A

Solution:

Let $I = \int 1 \cdot (\log 2x)^3 dx$

Using by part

$$=x(\log 2x)^3 - \int \left(3(\log 2x)^2 \cdot \frac{1}{x} \cdot x\right) dx$$
$$=x(\log 2x)^3 - 3\int (\log 2x)^2 dx$$
$$=(x\log 2x)^3 - 3 \cdot x(\log 2x)^2 + 3\int \left(x \cdot 2 \cdot \log 2x \cdot \frac{1}{x}\right) dx$$
$$=x\left[(\log 2x)^3 - 3(\log 2x)^2\right] + 6\int (\log 2x) dx$$
$$=x\left[(\log 2x)^3 - 3(\log 2x)^2\right] + 6x\log 2x - 6\int dx$$
$$=x\left[(\log 2x)^3 - 3(\log 2x)^2 + 6\log 2x - 6\right] + C$$

Question9

$$\int \frac{x+1}{(x-2)\sqrt{1-x}} dx =$$

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Options:

A.

$\log(x+1) - \log(x-2)\sqrt{1-x} + C$

B.

$\log(x-2)\sqrt{1-x} + C$

C.

$6\tan^{-1}\sqrt{1-x} - 2\sqrt{1-x} + C$

D.

$4\tan^{-1}\sqrt{1-x} - 2\sqrt{1-x} + C$

Answer: C

Solution:

Let $I=\int \frac{x+1}{(x-2\sqrt{1-x}}dx$

$\sqrt{1-x}=t$

$\Rightarrow \quad 1-x=t^2$

$\Rightarrow \quad x=1-t^2$

and $\quad \frac{-1}{2\sqrt{1-x}}dx=dt$

$\Rightarrow \quad \frac{dx}{\sqrt{1-x}}=-2dt$

$\therefore \quad I=\int \frac{1-t^2+1}{(1-t^2-2)}(-2)dt=-2\int \frac{t^2-2}{t^2+1}dt$

$=-2\int\left(\frac{t^2+1}{t^2+1}-\frac{3}{t^2+1}\right)dx$

$=-2\left(t-3\tan^{-1}t\right)+C$

$=-2\sqrt{1-x}+6\tan^{-1}\sqrt{1-x}+C$

$=6\tan^{-1}\sqrt{1-x}-2\sqrt{1-x}+C$

Question10

$\int \frac{1}{1+x+x^2}dx=$

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Options:

A.

$\frac{2}{\sqrt{3}}\log\left(\frac{2x+1+\sqrt{3}}{2x-1-\sqrt{3}}\right)+C$

B.

$\frac{1}{\sqrt{3}}\log\left(\frac{2x+1-\sqrt{3}}{2x+1+\sqrt{3}}\right)+C$

C.

$\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+C$

D.

$\frac{2}{\sqrt{5}}\tan^{-1}\left(\frac{2x+1}{\sqrt{5}}\right)+C$

Answer: C

Solution:

$$
\begin{aligned}
Let\ I &= \int \frac{1}{x^2+x+1} dx \\
&= \int \frac{1}{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dx \\
&= \frac{1}{\frac{\sqrt{3}}{2}} \tan^{-1}\left(\frac{x+1/2}{\sqrt{3}/2}\right) + C \\
&= \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + C
\end{aligned}
$$

Question11

If $\int \frac{dx}{(x \tan x+1)^2} = f(x) + C$**, then** $\lim_{x \to \frac{\pi}{2}} f(x) =$

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Options:

A.

$\frac{\pi}{2}$

B.

$\frac{2}{\pi}$

C.

$\frac{1}{\pi}$

D.

∞

Answer: B

Solution:

Let $I = \int \frac{dx}{(x\tan x+1)^2}$

$= \int \frac{\cos^2 x}{(x\sin x+\cos x)^2}dx$

$= \int \left(\frac{\cos x}{x}\right)\frac{x\cos x}{(x\sin x+\cos x)^2}dx$

$= \frac{\cos x}{x}\int \frac{x\cos x}{(x\sin x+\cos x)^2}$

$- \int \frac{(-x\sin x-\cos x)}{x^2}$

$\left(\int \frac{x\cos x}{(x\sin x+\cos x)^2}dx\right)$

$= \frac{\cos x}{x}\left(\frac{-1}{x\sin x+\cos x}\right) - \int \frac{x\sin x+\cos x}{x^2}\cdot\frac{1}{(x\sin x+\cos x)}dx$

$= \frac{-\cos x}{x(x\sin x+\cos x)}dx$

$\therefore f(x) = \frac{1}{x} + C$

$\therefore \lim_{x\to\frac{\pi}{2}} f(x) = \lim_{x\to\frac{\pi}{2}} \frac{-\cos x}{x(x\sin x+\cos x)} + \frac{1}{x}$

$= \frac{2}{\pi}$

Question12

$\int \sin^3 x \cos^2 x dx =$

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Options:

A.

$\frac{\sin^4 x\cos x}{5} - \frac{\sin^2 x\cos x}{15} - \frac{2\cos x}{15} + C$

B.

$-\frac{\sin^4 x\cos x}{5} - \frac{\sin^2 x\cos x}{15} + \frac{2\cos x}{15} + C$

C.

$\frac{\sin^4 x\cos' x}{5} - \frac{\sin^2 x\cos x}{15} + \frac{2x}{15} + C$

D.

$\frac{\sin^4 x\cos x}{5} + \frac{\sin^2 x\cos x}{3} - \frac{2x}{15} + C$

Answer: A

Solution:

$$I=\int \sin^3 x \cos^2 x dx$$
Let $\cos x = t$
$\Rightarrow \quad -\sin x dx = dt$
$$\therefore \quad I=\int (1-t^2)t^2(-dt)$$
$$=\int (t^4-t^2)dt=\frac{t^5}{5}-\frac{t^3}{3}+C$$
$$=\frac{\cos^5 x}{5}-\frac{\cos^3 x}{3}+C$$
$$=\frac{\cos x\left(1-\sin^2 x\right)^2}{5}-\frac{\cos x\left(1-\sin^2 x\right)}{3}+C$$
$$=\frac{\sin^4 x\cos x}{5}-\frac{2\sin^2 x\cos x}{5}+\frac{\cos x}{5}-\frac{\cos x}{3}+\frac{\cos x\sin^2 x}{3}+C$$
$$=\frac{\sin^4 x\cdot\cos x}{5}-\frac{\sin^2 x\cos x}{15}-\frac{2\cos x}{15}+C$$

Question13

If $\frac{3x^3-7x+1}{(x-2)^5}=\frac{A}{x-2}+\frac{B}{(x-2)^2}+\frac{C}{(x-2)^3}+\frac{D}{(x-2)^4}+\frac{E}{(x-2)^5}$, then $A(B+C+D+E)=$

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Options:

A.

0

B.

348

C.

64

D.

256

Answer: A

Solution:

Given, $\frac{3x^3-7x+1}{(x-2)^5}$
$$=\frac{A}{(x-2)}+\frac{B}{(x-2)^2}+\frac{C}{(x-2)^3}+\frac{D}{(x-2)^4}+\frac{E}{(x-2)^5}$$
$$=\frac{A(x-2)^4+B(x-2)^3+C(x-2)^2+D(x-2)+E}{(x-2)^5}$$
$$=Ax^4+(-8A+B)x^3+(24A-6B+C)x^2+(-32A+12B-4C+D)x+16A-8B+4C-2D+E$$

Comparing the coefficient, we get

$$A=0$$
$$-8A+B=3$$
$$\Rightarrow \quad B=3+8A=3$$
$$24A-6B+C=0$$
$$\Rightarrow \quad C=6B-24A=18-0=18$$
$$-32A+12B-4C+D=-7$$
$$\Rightarrow \quad D=-7+32A-12B+4C=-7+0-36+72=29$$
$$16A-8B+4C-2D+E=1$$
$$\Rightarrow \quad E=1-16A+8B-4C+2D$$
$$=1-0+24-72+58=11$$
$$\therefore \quad A(B+C+D+E)=0(3+18+29+11)=0$$

Question14

$$\int(\sqrt{\tan x}+\sqrt{\cot x})dx=$$

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Options:

A.

$2\tan^{-1}\left(\frac{\tan x-1}{\sqrt{\tan x}}\right)+C$

B.

$\tan^{-1}\left(\frac{\tan x-2}{2\sqrt{\tan x}}\right)+C$

C.

$\sqrt{2}\tan^{-1}\left(\frac{\tan x-1}{\sqrt{2\tan x}}\right)+C$

D.

$\sqrt{2}\tan^{-1}\left(\frac{\tan x+1}{\sqrt{2\tan x}}\right)+C$

Answer: C

Solution:

$$(\sqrt{\tan x}+\sqrt{\cot x})dx$$
$$=\int\left(\sqrt{\tan x}+\frac{1}{\sqrt{\tan x}}\right)dx$$
$$=\int\frac{\tan x+1}{\sqrt{\tan x}}dx$$

Let $u=\sqrt{\tan x}$, then $u^2=\tan x$ and

$$
\begin{aligned}
2u\,du &= \sec^2 x\,dx \\
&= \left(1+\tan^2 x\right)dx \\
&= \left(1+u^4\right)dx \\
&= \int \frac{u^2+1}{u}\cdot\frac{2u}{1+u^4}du \\
&= \int \frac{2\left(u^2+1\right)}{1+u^4}du = \int \frac{2\left(1+\frac{1}{u^2}\right)}{\frac{1}{u^2}+u^2}du \\
&= \int \frac{2\left(1+\frac{1}{u^2}\right)}{\left(u-\frac{1}{u}\right)^2+2}du
\end{aligned}
$$

Again, Let $v = u - \dfrac{1}{u}$

$$
\begin{aligned}
\Rightarrow \quad dv &= \left(1+\frac{1}{u^2}\right)du \\
&= \int \frac{2}{2\left(\frac{v^2}{2}+1\right)}dv = \int \frac{1}{\left(\frac{v^2}{2}+1\right)}dv \\
&= \int \frac{1}{\left(\frac{v}{\sqrt{2}}\right)^2+1}dv \\
&= \tan^{-1}\left(\frac{v}{\sqrt{2}}\right)\cdot\sqrt{2}+C
\end{aligned}
$$

where $c =$ constant

$$
\begin{aligned}
&= \sqrt{2}\tan^{-1}\left(\frac{u-\frac{1}{u}}{\sqrt{2}}\right)+C \\
&= \sqrt{2}\tan^{-1}\left(\frac{\sqrt{\tan x}-\frac{1}{\sqrt{\tan x}}}{\sqrt{2}}\right)+C \\
&= \sqrt{2}\tan^{-1}\left(\frac{\tan x-1}{\sqrt{2\tan x}}\right)+C
\end{aligned}
$$

Question15

$\int \frac{\sqrt{x-2}}{2x+4}dx =$

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Options:

A.

$\sqrt{x-2}-\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{x-2}}{2}\right)+C$

B.

$\sqrt{x-2}-2\tan^{-1}\left(\frac{\sqrt{x-2}}{2}\right)+C$

C.

$\sqrt{x-2}+2\tan^{-1}\left(\frac{\sqrt{x-2}}{2}\right)+C$

D.

$\sqrt{x-2}+\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{x-2}}{2}\right)+C$

Answer: B

Solution:

Let $I=\int \frac{\sqrt{x-2}}{2x+4}dx$

and $u=\sqrt{x-2}$

$\Rightarrow u^2=x-2$ and $x=u^2+2$

Differentiate x w.r.t u, we get

$\frac{dx}{du}=2u \Rightarrow dx=2udu$

$$I=\int \frac{\sqrt{x-2}}{2x+4}dx=\int \frac{2u\cdot u}{2\left(u^2+2\right)+4}du$$
$$=\int \frac{2u^2}{2u^2+8}du=\int \frac{u^2}{u^2+4}du$$
$$=\int \frac{u^2+4-4}{u^2+4}du=\int\left(1-\frac{4}{u^2+4}\right)du$$
$$=u-4\cdot\frac{1}{2}\cdot\tan^{-1}\left(\frac{u}{2}\right)+C$$

where, $C=$ constant

$$=u-2\tan^{-1}\left(\frac{u}{2}\right)+C$$
$$=\sqrt{x-2}-2\tan^{-1}\left(\frac{\sqrt{x-2}}{2}\right)+C$$

Question16

If $\int x^{49}\left[\tan^{-1}x^{50}+\frac{x^{50}}{1+x^{100}}\right]dx=\frac{x^n}{k}f(x)+c$, then

$f(x)-f\left(\sqrt[k]{x^n}\right)=$

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Options:

A.

$k+n$

B.

$k-n$

C.

$1/k$

D.

$1/n$

Answer: B

Solution:

Given,

$$\begin{aligned} I &= \int x^{49}\left[\tan^{-1}\left(x^{50}\right)+\frac{x^{50}}{1+x^{100}}\right]dx \\ &= \frac{x^n}{k}f(x)+c \end{aligned}$$

Let $u=x^{50}$, then $du=50x^{49}dx$

$$\begin{aligned} so, I &= \int x^{49}\left[\tan^{-1}x^{50}+\frac{x^{50}}{1+x^{100}}\right]dx \\ &= \int x^{49}\left[\tan^{-1}(u)+\frac{u}{1+u^2}\right]\cdot\frac{du}{50x^{49}} \end{aligned}$$

$$\begin{aligned} &= \frac{1}{50}\int\left(\tan^{-1}(u)+\frac{u}{1+u^2}\right)du \\ &= \frac{1}{50}\left[\int\tan^{-1}(u)du+\int\frac{u}{1+u^2}du\right] \\ &= \frac{1}{50}\left[u\tan^{-1}(u)-\int\frac{u}{1+u^2}du+\int\frac{u}{1+u^2}du\right] \end{aligned}$$

[using product rule]

$$\begin{aligned} &= \frac{1}{50}\left[u\tan^{-1}(u)\right]+c \\ &= \frac{1}{50}\cdot x^{50}\cdot\tan^{-1}\left(x^{50}\right)+c \end{aligned}$$

Comparing to original equation, we get

$n=50, k=50, f(x)=\tan^{-1}\left(x^{50}\right)$

Now, $f(x)-f\left(\sqrt[k]{x^n}\right)$

$$\begin{aligned} &= f(x)-f\left(x^{n/k}\right) \\ &= \tan^{-1}\left(x^{50}\right)-f\left(x^{50/50}\right) \\ &= \tan^{-1}\left(x^{50}\right)-f(x) \\ &= \tan^{-1}\left(x^{50}\right)-\tan^{-1}\left(x^{50}\right)=0 \end{aligned}$$

$$\therefore f(x)-f\left(\sqrt[k]{x^n}\right)=0 \qquad \ldots(i)$$

Now, $k-n=50-50=0 \qquad \ldots(ii)$

From Eqs. (i) and (ii), we get

$$f(x)-f\left(\sqrt[k]{x^n}\right)=k-n$$

Question17

$\int \frac{x}{\sqrt{x^2-2x+5}} dx =$

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Options:

A.

$\sqrt{x^2-2x+5}+\sinh^{-1}\left(\frac{x-1}{2}\right)+C$

B.

$\frac{1}{2}\sqrt{x^2-2x+5}+\sin^{-1}\left(\frac{x-1}{2}\right)+C$

C.

$2\sqrt{x^2-2x+5}+\cosh^{-1}\left(\frac{x-1}{2}\right)+C$

D.

$\sqrt{x^2-2x+5}-\cos^{-1}\left(\frac{x-1}{2}\right)+C$

Answer: A

Solution:

$$\text{Let } I=\int \frac{x}{\sqrt{x^2-2x+5}}dx$$
$$=\int \frac{x-1+1}{\sqrt{x^2-2x+5}}dx$$
$$=\int \frac{x-1}{\sqrt{x^2-2x+5}}+\int \frac{1}{\sqrt{x^2+2x+5}}dx$$

Let $u=x^2-2x+5$, then $du=(2x-2)dx$

$$=2(x-1)dx$$
$$=\frac{1}{2}\int \frac{du}{\sqrt{u}}+\int \frac{1}{\sqrt{(x-1)^2+4}}dx$$
$$=\frac{1}{2}\left(2u^{\frac{1}{2}}\right)+\sin h^{-1}\frac{(x-1)}{2}+C$$
$$\left[\because \int \frac{1}{\sqrt{x^2+a^2}}=\sin h^{-1}\frac{x}{a}\right]$$

Where, $C=$ constant

$$=\sqrt{u}+\sinh^{-1}\left(\frac{(x-1)}{2}\right)+C$$
$$=\sqrt{x^2-2x+5}+\sinh^{-1}\left(\frac{x-1}{2}\right)+C$$

Question18

For $0 < x < 1, \int \left[\tan^{-1}\left(1 - x + x^2\right) + \tan^{-1}(1 - x)\right] dx =$

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Options:

A.

$x \cot^{-1} x + \log \sqrt{1 + x^2} + C$

B.

$x \tan^{-1} x - \log\left(1 + x^2\right) + C$

C.

$x \cot^{-1} x + \frac{3}{4} \log\left(1 + x^2\right) + C$

D.

$x \tan^{-1} x - \frac{3}{4} \log \sqrt{1 + x^2} + C$

Answer: A

Solution:

For $0 < x < 1$,

$$I = \int \left[\tan^{-1}\left(1 - x + x^2\right) + \tan^{-1}(1 - x)\right] dx$$

$$= \int \tan^{-1}\left[\frac{\left(1 - x + x^2\right) + (1 - x)}{1 - \left(1 - x + x^2\right)(1 - x)}\right] dx$$

$$= \int \tan^{-1}\left(\frac{2 - 2x + x^2}{x\left(2 - 2x + x^2\right)}\right) dx$$

$$= \int \tan^{-1}\left(\frac{1}{x}\right) dx$$

$$= \int \cot^{-1}(x) dx = \int \left(\frac{\pi}{2} - \tan^{-1} x\right) dx$$

$$= \int \frac{\pi}{2} dx - \int \tan^{-1} x dx$$

$$= \frac{\pi}{2} x - \left[x \tan^{-1} x - \frac{1}{2} \ln\left(1 + x^2\right) + C\right]$$

[Using product rule]

$= \frac{\pi}{2} x - x \tan^{-1} x + \frac{1}{2} \ln\left(1 + x^2\right) + C$

where $C =$ constant of integration

$$= x\left(\frac{\pi}{2} - \tan^{-1} x\right) + \frac{1}{2}\ln\left(1 + x^2\right) + C$$
$$= x\cot^{-1} x + \frac{1}{2}\ln\left(1 + x^2\right) + C$$
$$= x\cot^{-1} x + \log\sqrt{1 + x^2} + C$$

Question19

If $\frac{3x+1}{(x-1)(x^2+2)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+2}$**, then** $5(A-B) =$

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Options:

A.

$A + C$

B.

8 C

C.

$C + 8$

D.

$\frac{C}{8}$

Answer: B

Solution:

$$\because \frac{3x+1}{(x-1)\left(x^2+2\right)} = \frac{A}{x-1} + \frac{Bx+c}{x^2+2}$$
$$\Rightarrow 3x + 1 = A\left(x^2 + 2\right) + (Bx + C)(x - 1)$$
$$\Rightarrow 3x + 1 = (A + B)x^2 + (C - B)x + (2A - C)$$

Comparing both sides, we get

$$A + B = 0$$
$$\Rightarrow \quad A = -B \quad \ldots(i)$$

and $C - B = 3$ $\quad \ldots(ii)$

and $2A - C = 1$ $\quad \ldots(iii)$

put the value of A from Eq. (i) to Eq. (iii)

$$\Rightarrow \quad -2B - C = 1$$
$$\Rightarrow \quad C = -2B - 1 \quad \ldots(iv)$$

By Eqs. (ii) and (iv),

We get

$(-2B-1)-B=3 \Rightarrow B=-4/3$

and $A=-(-4/3)=4/3$

$$\begin{aligned} Also,\ C &= -2B-1=-2(-4/3)-1 \\ &= \frac{8}{3}-1=5/3 \end{aligned}$$

$$\begin{aligned} thus,\ 5(A-B) &= 5\left(\frac{4}{3}-(-4/3)\right) \\ &= 5\left(\frac{8}{3}\right)=\frac{40}{3} \end{aligned}$$

and $8C=8\times\frac{5}{3}=\frac{40}{3}$

Therefore, $5(A-B)=8C$

Question20

$$\int \frac{\sec^2 x}{\sin^7 x} dx-\int \frac{7}{\sin^7 x} dx=$$

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Options:

A.

$\frac{1}{\sin^6 x \cos x}+C$

B.

$\frac{\tan x}{\sin^8 x}+C$

C.

$\sin^8 x \cos x+C$

D.

$\sec x \tan^7 x+C$

Answer: A

Solution:

Let $I=\int \frac{\sec^2 x-7}{\sin^7 x} dx$

$$\Rightarrow \quad I=\int \frac{1-7\cos^2 x}{\sin^7 x \cos^2 x} dx$$

$$\Rightarrow \quad I=\int \frac{\sin^5 x\left(1-7\cos^2 x\right) dx}{\sin^{12} x \cos^2 x}$$

Put $\sin^6 x \cos x = t$

$$\left(6 \sin^5 x \cos^2 x - \sin^7 x\right) dx = dt$$
$$\Rightarrow \sin^5 x \left(6 \cos^2 x - \sin^2 x\right) dx = dt$$
$$\Rightarrow \sin^5 x \left(6 \cos^2 x - 1 + \cos^2 x\right) dx = dt$$
$$\Rightarrow \sin^5 x \left(1 - 7 \cos^2 x\right) dx = -dt$$
$$\therefore I = \int \frac{-dt}{t^2}$$
$$\Rightarrow I = \frac{1}{t} + C$$
$$\Rightarrow I = \frac{1}{\sin^6 x \cos x} + C$$

Question21

If $\int \left(x^6 + x^4 + x^2\right) \sqrt{2x^4 + 3x^2 + 6} dx = f(x) + c$, then $f(3) =$

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Options:

A.

$\frac{3}{2}(95)^{3/2}$

B.

$\frac{3}{2}(195)^{3/2}$

C.

$\frac{3}{2}(265)^{3/2}$

D.

$\frac{3}{2}(175)^{3/2}$

Answer: B

Solution:

$$\int\left(x^6+x^4+x^2\right)\sqrt{2x^4+3x^2+6}dx$$
$$=\int x\left(x^5+x^3+x\right)\sqrt{2x^4+3x^2+6}dx$$
$$=\int\left(x^5+x^3+x\right)\sqrt{2x^6+3x^4+6x^2}dx$$

Put $2x^6+3x^4+6x^2=t$

$\Rightarrow 12\left(x^5+x^3+x\right)dx=dt$

$$=\frac{1}{12}\int\sqrt{t}\cdot dt=\frac{1}{12}\frac{t^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+C$$
$$=\frac{t^{\frac{3}{2}}}{18}+C=\frac{1}{18}\left(2x^6+3x^4+6x^2\right)^{\frac{3}{2}}+C$$

So, $f(x)=\dfrac{1}{18}\left(2x^6+3x^4+6x^2\right)^{3/2}$

$\Rightarrow f(3)=\dfrac{1}{18}(9(195))^{\frac{3}{2}}=\dfrac{3}{2}(195)^{\frac{3}{2}}$

Question22

$$\int\frac{dx}{(x+1)\sqrt{x^2+1}}=$$

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Options:

A.

$\frac{1}{\sqrt{2}}\sinh^{-1}\left(\frac{1+x}{1-x}\right)+C$

B.

$\frac{1}{\sqrt{2}}\sinh^{-1}\left(\frac{1-x}{1+x}\right)+C$

C.

$-\frac{1}{\sqrt{2}}\sinh^{-1}\left(\frac{1-x}{1+x}\right)+C$

D.

$-\frac{1}{\sqrt{2}}\sinh^{-1}\left(\frac{1+x}{1-x}\right)+C$

Answer: C

Solution:

Let $I=\int \frac{dx}{(x+1)\sqrt{x^2+1}}$

Put $x+1=\frac{1}{t} \Rightarrow dx=-\frac{1}{t^2}dt$

$x=\frac{1}{t}-1$

$\therefore \quad I=\int \frac{-\frac{1}{t^2}dt}{\frac{1}{t}\sqrt{\left(\frac{1}{t}-1\right)^2+1}}$

$I=\int \frac{-dt}{\sqrt{2t^2-2t+1}}$

$I=\frac{-1}{\sqrt{2}}\int \frac{dt}{\sqrt{\left(t-\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2}}$

$I=-\frac{1}{2}\sinh^{-1}2\left(t-\frac{1}{2}\right)+C$

$I=-\frac{1}{\sqrt{2}}\sinh^{-1}\left(\frac{1-x}{1+x}\right)+C$

Question23

If $\int \frac{dx}{2\cos x+3\sin x+4}=\frac{2}{\sqrt{3}}f(x)+C$, then $f\left(\frac{2\pi}{3}\right)=$

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Options:

A.

$\frac{\pi}{12}$

B.

$\frac{\pi}{8}$

C.

$\frac{5\pi}{12}$

D.

$\frac{5\pi}{8}$

Answer: C

Solution:

$$\int \frac{dx}{2\cos x + 3\sin x + 4}$$

Let $t = \tan\frac{x}{2} \Rightarrow \sin x = \frac{2t}{1+t^2}$

$\cos x = \frac{1-t^2}{1+t^2}$

and $dx = \frac{2dt}{1+t^2}$

$$= \int \frac{2dt}{(1+t^2)\left[\frac{2(1-t^2)}{(1+t^2)} + \frac{6t}{(1+t^2)} + 4\right]}$$

$$= \int \frac{2dt}{2 - 2t^2 + 6t + 4 + 4t^2}$$

$$= \int \frac{dt}{t^2 + 3t + 3} = \int \frac{dt}{\left(t + \frac{3}{2}\right)^2 + \frac{3}{4}}$$

put $u = t + \frac{3}{2} \Rightarrow du = dt$

$$= \int \frac{1}{u^2 + \left(\frac{\sqrt{3}}{2}\right)^2} du$$

$$= \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} \tan^{-1}\left(\frac{u}{\left(\frac{\sqrt{3}}{2}\right)}\right) + C$$

$$= \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2u}{\sqrt{3}}\right) + C$$

$$= \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2t+3}{\sqrt{3}}\right) + C$$

$$= \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2\tan\frac{x}{2} + 3}{\sqrt{3}}\right) + C$$

Thus, $f(x) = \tan^{-1}\left(\frac{2\tan\left(\frac{x}{2}\right) + 3}{\sqrt{3}}\right)$

$$\therefore f\left(\frac{2\pi}{3}\right) = \tan^{-1}\left(\frac{2\tan\left(\frac{\pi}{3}\right) + 3}{\sqrt{3}}\right)$$

$$= \tan^{-1}\left(\frac{2\sqrt{3} + 3}{\sqrt{3}}\right)$$

$$= \tan^{-1}(2 + \sqrt{3}) = \frac{5\pi}{12}$$

Question24

If $\int \frac{1}{((x+4)^3(x+1)^5)^{1/4}} dx = A \cdot \left(\frac{x+4}{x+1}\right)^n + C$

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Options:

A.

n A=3

B.

$n+\frac{1}{A}=-\frac{1}{2}$

C.

$A+n=1$

D.

$A=n$

Answer: B

Solution:

$$\because \int \frac{1}{\left((x+4)^3(x+1)^5\right)^{\frac{1}{4}}} dx$$
$$= A \cdot \left(\frac{x+4}{x+1}\right)^n + C$$
$$= \int \frac{1}{(x+1)^2\left(\frac{x+4}{x+1}\right)^{3/2}} dx$$
$$\text{Let } t=\frac{x+4}{x+1}$$
$$\Rightarrow dt=\frac{-3}{(x+1)^2} dx = -\int \frac{1}{3} t^{-3/4} dt$$
$$= -\frac{1}{3}\left[\frac{\frac{-3}{4}+1}{\frac{-3}{4}+1}\right] + C$$
$$= -\frac{1}{3}\left[4t^{1/4}\right] + C$$
$$= -\frac{4}{3}\left(\frac{x+4}{x+1}\right)^{1/4} + C$$

Comparing with $A=\frac{(x+4)^n}{x+1}+C$

$A=-\frac{4}{3}, n=\frac{1}{4}$

Therefore, $n+\frac{1}{A}=\frac{1}{4}+\frac{1}{\left(\frac{-4}{3}\right)}$

$$=\frac{1}{4}-\frac{3}{4}$$
$$=\frac{-2}{4}=\frac{-1}{2}$$

Question25

$\int \frac{x+1}{x^3-1} dx =$

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Options:

A.

$\frac{1}{3}\log\left(\frac{x+1}{x^2+x+1}\right)+C$

B.

$\frac{1}{3}\log\left(\frac{(x-1)^2}{x^2+x+1}\right)+C$

C.

$\frac{1}{3}\log\left(\frac{x-1}{x^2+x+1}\right)+C$

D.

$\frac{1}{3}\log\left(\frac{(x+1)^2}{x^2-x+1}\right)+C$

Answer: B

Solution:

Let $I=\int\frac{x+1}{x^3-1}dx$

$$=\int\frac{2}{3(x-1)}dx+\int\frac{-2x-1}{3\left(x^2+x+1\right)}dx$$
$$=\frac{2}{3}\ln|x-1|-\frac{1}{3}\ln\left|x^2+x+1\right|+C$$

where C is constant of integration.

$=\frac{1}{3}\ln\left|\frac{(x-1)^2}{x^2+x+1}\right|+C$

Question26

$$\int\frac{x^4-16x^2+2x+8}{x^3-4x^2+2}dx=$$

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Options:

A.

$\frac{x^2+8x+C}{2}$

B.

x^2+8x+C

C.

x^3-4x+C

D.

$\frac{x^2-8x+C}{2}$

Answer: A

Solution:

$$\text{Let } I=\int\frac{x^4-16x^2+2x+8}{x^3-4x^2+2}dx$$
$$=\int\frac{(x+4)\left(x^3-4x^2+4\right)}{(x^3-4x^2+2}dx$$
$$=\int(x+4)dx=\frac{x^2}{2}+4x+C_1$$
$$=\frac{x^2+8x+2C_1}{2}$$
$$=\frac{x^2+8x+C}{2}$$

where C_1 is constant of integration $C=2C_1$.

Question27

$$\int\frac{\sec^2 x}{(\sec x+\tan x)^{\frac{5}{2}}}dx=$$

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Options:

A.

$-\frac{(\sec x+\tan x)^{\frac{5}{2}}}{5}-\frac{(\sec x+\tan x)^{\frac{7}{2}}}{7}+C$

B.

$-\frac{(\sec x-\tan x)^{\frac{5}{2}}}{5}-\frac{(\sec x-\tan x)^{\frac{7}{2}}}{7}+C$

C.

$-\frac{(\sec x+\tan x)^{\frac{3}{2}}}{3}-\frac{(\sec x+\tan x)^{\frac{7}{2}}}{7}+C$

D.

$-\frac{(\sec x-\tan x)^{\frac{3}{2}}}{3}-\frac{(\sec x-\tan x)^{\frac{7}{2}}}{7}+C$

Answer: D

Solution:

Let $I=\int \frac{\sec^2 x}{(\sec x+\tan x)^{\frac{5}{2}}} dx$

Put $\sec x+\tan x=t$

$\therefore \sec x-\tan x=\frac{1}{t}$

$\therefore \sec x=\frac{1}{2}\left(t+\frac{1}{t}\right)$ and $\tan x=\frac{1}{2}\left(t-\frac{1}{t}\right)$

Differentiate w.r.t. x, we get

$$\Rightarrow \quad \left(\sec x \tan x+\sec^2 x\right) dx=dt$$
$$\Rightarrow \quad dx=\frac{dt}{\sec x \tan x+\sec^2 x}$$
$$I=\int \frac{\sec^2 x}{t^{\frac{5}{2}}} \times \frac{dt}{\sec x \tan x+\sec^2 x}$$
$$=\int \frac{\sec x}{t^{\frac{5}{2}}} \times \frac{dt}{(\sec x+\tan x)}$$
$$=\int\left(\frac{1}{2}\left(t+\frac{1}{t}\right)\right) \cdot \frac{1}{t^{\frac{5}{2}}} \times \frac{1}{t} dt$$
$$=\frac{1}{2} \int\left(t+\frac{1}{t}\right) t^{\frac{-7}{2}} dt$$
$$=\frac{1}{2}\left[\int t^{\frac{-5}{2}} dt+\int t^{\frac{-9}{2}} dt\right]$$
$$=\frac{1}{2}\left[\frac{t^{\frac{-3}{2}}}{\frac{-3}{2}}+\frac{t^{\frac{-7}{2}}}{\frac{-7}{2}}\right]+C$$
$$=\frac{1}{2}\left[\frac{-2}{3} \times \frac{1}{t^{\frac{3}{2}}}-\frac{2}{7} \frac{1}{\frac{7}{2}}\right]+C$$
$$=\frac{-1}{3}(\sec x-\tan x)^{\frac{3}{2}}-\frac{1}{7}(\sec x-\tan x)^{\frac{7}{2}}+C$$

Question28

$\int \frac{1}{\cos x}\left[\frac{1}{\sin x}-\frac{1}{\sin x+3 \cos x}\right] dx=$

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Options:

A.

$\frac{1}{3} \log \left|\frac{\sin x}{\sin x+3 \cos x}\right|+C$

B.

$\log \left|\frac{\cos x}{\sin x+3 \cos x}\right|+c$

C.

$\frac{1}{3} \log \left|\frac{\cos x}{\sin x+3 \cos x}\right|+C$

D.

$\log\left|\frac{\sin x}{\sin x+3\cos x}\right|+c$

Answer: D

Solution:

$$\begin{aligned}
Let\ I &= \int \frac{1}{\cos x}\left[\frac{1}{\sin x}-\frac{1}{\sin x+3\cos x}\right]dx \\
&= \int \frac{2}{\sin 2x}dx - \int \frac{\sec^2 x}{\tan x+3}dx \\
&= 2\int \operatorname{cosec} 2x dx - \int \frac{dz}{z+3},\ \text{where}
\end{aligned}$$

$$\begin{aligned}
z &= \tan x \\
&= \frac{2\log_e|\operatorname{cosec} 2x-\cot 2x|}{2} - \ln|z+3|+C \\
&= \ln|\operatorname{cosec} 2x-\cot 2x| - \ln|\tan x+3|+C
\end{aligned}$$

$$= \ln\left|\frac{\frac{1}{\sin 2x}-\frac{\cos 2x}{\sin 2x}}{\frac{\sin x}{\cos x}+3}\right|+C$$

$$= \ln\left|\frac{\frac{1-\cos 2x}{\sin 2x}}{\frac{\sin x+3\cos x}{\cos x}}\right|+C$$

$$= \ln\left|\frac{2\sin^2 x}{2\sin x\cdot\cos x}\times\frac{\cos x}{\sin x+3\cos x}\right|+C$$

$$= \ln\left|\frac{\sin x}{\sin x+3\cos x}\right|+C$$

Question29

$\int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx =$

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Options:

A.

$2\left[x\tan^{-1}x-\log\sqrt{1+x^2}\right]+C$

B.

$2x\tan^{-1}x+\log\sqrt{1+x^2}+C$

C.

$x\tan^{-1}x+\log\sqrt{1-x^2}+C$

D.

$2\left[\tan^{-1}x-\log\sqrt{1+x^2}\right]+C$

Answer: A

Solution:

$$\begin{aligned}&\text{Let } I=\int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) dx\\&=\int 2\tan^{-1} x dx=2\int\left(\tan^{-1} x\right)\cdot 1 dx\end{aligned}$$

(I) (II)

$$\begin{aligned}&=2\left[x\tan^{-1} x-\int \frac{x}{1+x^2} dx\right]\\&=2\left[x\tan^{-1} x-\frac{1}{2}\int \frac{2x}{1+x^2} dx\right]\\&=2\left[x\tan^{-1} x-\ln\left(1+x^2\right)\right]+C\end{aligned}$$

Question30

If $\frac{ax+5}{(x^2+b)(x+3)}=\frac{x+21}{12(x^2+b)}+\frac{c}{12(x+3)}$, then $b^2=$

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Options:

A.

a^3-c

B.

a^2+c

C.

$a-c$

D.

$a+c$

Answer: A

Solution:

$$\begin{aligned}&\frac{ax+5}{(x^2+b)(x+3)}=\frac{x+21}{12\left(x^2+b\right)}+\frac{c}{12(x+3)}\\&\Rightarrow 12(ax+5)=(x+21)(x+3)+c\left(x^2+b\right)\\&\Rightarrow 12ax+60=x^2+3x+21x+63+cx^2+bc\\&\Rightarrow 12ax+60=(1+c)x^2+24x+63+bc\end{aligned}$$

Comparing coefficient

$$\begin{aligned} \Rightarrow 1+c &= 0 \quad & 12a &= 24 \quad & 63+bc &= 60 \\ c &= -1 & a &= 2 & 63-b &= 60 \end{aligned}$$

$$\Rightarrow \quad b=3$$
$$b^2 = 9 = a^3 - c$$
$$\therefore \quad b^2 = a^3 - c$$

Question31

$\int \left(\sum_{r=0}^{\infty} \frac{x^r 2^r}{r!}\right) dx =$

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Options:

A.

$e^x + C$

B.

$\frac{-2}{1-2x} + C$

C.

$2e^{2x} + C$

D.

$\frac{e^{2x}}{2} + C$

Answer: D

Solution:

$$\begin{aligned} & \int\left(\sum_{r=0}^{\infty} \frac{x^r 2^r}{r!}\right) dx \\ & =\int\left(1+\frac{(2x)^1}{1!}+\frac{(2x)^2}{2!}+\frac{(2x)^3}{3!}+\ldots\right) dx \\ & =\int e^{2x} dx=\frac{e^{2x}}{2}+C \end{aligned}$$

Question32

$\int \frac{dx}{12\cos x + 5\sin x} =$

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Options:

A.

$\frac{1}{13}\log\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}-\frac{1}{2}\tan^{-1}\frac{5}{12}\right)\right|+C$

B.

$\frac{5}{12}\log\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}-\frac{1}{2}\tan^{-1}\frac{5}{12}\right)\right|+C$

C.

$\frac{1}{13}\log\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}+\frac{1}{2}\tan^{-1}\frac{5}{12}\right)\right|+C$

D.

$\frac{5}{12}\log\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}+\frac{1}{2}\tan^{-1}\frac{5}{12}\right)\right|+C$

Answer: A

Solution:

$$\int \frac{1}{12\cos x+5\sin x}dx$$
$$R=\sqrt{12^2+5^2}=\sqrt{169}=13$$
$$\tan\alpha=\frac{5}{12}$$
$$=\int \frac{1}{13\left(\frac{12}{13}\cos x+\frac{5}{13}\sin x\right)}dx$$
$$=\frac{1}{13}\int \frac{1}{\cos(x-\alpha)}dx$$
$$=\frac{1}{13}\log\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}-\frac{1}{2}\tan^{-1}\frac{5}{12}\right)\right|+C$$

Question33

If $\int \frac{\cos^3 x}{\sin^2 x+\sin^4 x}dx = c-\operatorname{cosec} x - f(x)$**, then** $f\left(\frac{\pi}{2}\right)=$

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Options:

A.

1

B.

0

C.

$\pi/2$

D.

π

Answer: C

Solution:

$$\int \frac{\cos^3 x}{\sin^2 x+\sin^4 x} dx$$
$$=\int \frac{\cos x\left(1-\sin^2 x\right)}{\sin^2 x\left(1+\sin^2 x\right)} dx$$

Put $u=\sin x$

$$\begin{aligned} du=\cos x dx & =\int \frac{1-u^2}{u^2\left(1+u^2\right)} du \\ & =\int\left(\frac{1}{u^2}-\frac{2}{1+u^2}\right) du \\ & =-\frac{1}{u}-2 \tan ^{-1}(u)+C \\ & =-\frac{1}{\sin x}-2 \tan ^{-1}(\sin x)+C \end{aligned}$$
$$\Rightarrow f(x)=2 \tan ^{-1}(\sin x)$$
$$\Rightarrow f\left(\frac{\pi}{2}\right)=2 \tan ^{-1}(1)=2 \times \frac{\pi}{4}=\frac{\pi}{2}$$

Question34

$$\int \frac{13 \cos 2x-9 \sin 2x}{3 \cos 2x-4 \sin 2x} dx=$$

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Options:

A.

$3x-\frac{1}{2} \log |3 \cos 2x-4 \sin 2x|+C$

B.

$\frac{x}{2}-3 \log |3 \cos 2x-4 \sin 2x|+C$

C.

$3x+\frac{1}{2}\log|3\cos 2x-4\sin 2x|+C$

D.

$x+\frac{3}{2}\log|3\cos 2x-4\sin 2x|+C$

Answer: A

Solution:

$$\begin{aligned}&\int \frac{13\cos 2x-9\sin 2x}{3\cos 2x-4\sin 2x}dx\\ =&\int\left(3-\frac{1(-6\sin 2x-8\cos 2x)}{2(3\cos 2x-4\sin 2x)}\right)dx\\ =&3x-\frac{1}{2}\log|3\cos 2x-4\sin 2x|+C\end{aligned}$$

Question35

$\int\sqrt{x^2+x+1}dx$

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Options:

A.

$\frac{(2x+1)}{4}\sqrt{x^2+x+1}+\frac{3}{8}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+C$

B.

$\frac{x+1}{4}\sqrt{x^2+x+1}+\frac{3}{8}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+C$

C.

$\frac{x+1}{4}\sqrt{x^2+x+1}-\frac{3}{8}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+C$

D.

$\frac{(2x+1)}{4}\sqrt{x^2+x+1}-\frac{3}{8}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+C$

Answer: A

Solution:

$$\int \sqrt{x^2+x+1}dx$$
$$= \int \sqrt{x^2+x+\frac{1}{4}-\frac{1}{4}+1}dx$$
$$= \int \sqrt{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}dx$$
$$= \int \sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dx$$
$$= \frac{x+\frac{1}{2}}{2}\sqrt{x^2+x+1}+\frac{3}{8}\ln\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right|+C$$
$$= \frac{2x+1}{4}\sqrt{x^2+x+1}+\frac{3}{8}\sin^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+C$$

Question36

If $\frac{x+1}{(x-1)^2(x^2+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+1}$**, then** $\sqrt{3A^2+4D^2+5C^2+B^2} =$

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Options:

A.

$\frac{3}{2}$

B.

$\frac{1}{2}$

C.

1

D.

2

Answer: D

Solution:

$$\frac{x+1}{(x-1)^2(x^2+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+1}$$
$$x+1 = A(x-1)\left(x^2+1\right)+B\left(x^2+1\right)+(Cx+D)(x-1)^2 \qquad \ldots(i)$$

Put $x=1$

$2=2B \Rightarrow B=1$

In Eq. (i) put $x=0$

$$1 = -A + B + D \quad \ldots(ii)$$
$$\Rightarrow \quad A = D \quad \ldots(ii)$$

In Eq. (i) put $x = -1$

$$0 = -4A + 2 - 4C + 4D$$
$$4C = 2 \Rightarrow C = \frac{1}{2} \quad \ldots(iii)$$

In Eq. (i) put $x = 2$

$$3 = 5A + 5 + 1 + D \Rightarrow 6A = -3$$

$$A = -\frac{1}{2} = D$$
$$\sqrt{3A^2 + 4D^2 + 5C^2 + B^2}$$

$$= \sqrt{\frac{3}{4} + \frac{4}{4} + \frac{5}{4} + 1}$$
$$= \sqrt{\frac{16}{4}} = 2$$

Question37

$$\int \frac{1}{9\cos^2 x - 24\sin x \cos x + 16\sin^2 x} dx =$$

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Options:

A.

$\frac{\cos x}{4(3\cos x - 4\sin x)} + C$

B.

$\frac{\sin x}{4(3\cos x - 4\sin x)} + C$

C.

$\frac{\cos x}{3\cos x - 4\sin x} + C$

D.

$\frac{\sin x}{3\cos x - 4\sin x} + C$

Answer: A

Solution:

$$I=\int \frac{1}{9\cos^2 x-24\sin x\cos x+16\sin^2 x}dx=$$

$$\Rightarrow I=\int \frac{1}{(3\cos x-4\sin x)^2}dx$$

$$\Rightarrow I=\int \frac{\sec^2 x}{(3-4\tan x)^2}dx$$

$$3-4\tan x=t$$

$$-4\sec^2 x dx=dt$$

$$\Rightarrow I=\frac{1}{-4}\int t^{-2}dt$$

$$=\frac{1}{4}\cdot\frac{1}{t}+C=\frac{\cos x}{4(3\cos x-4\sin x)}+C$$

Question38

If $\int \frac{1}{\cot\frac{x}{2}\cot\frac{x}{3}\cot\frac{x}{6}}dx=A\log\left|\cos\frac{x}{2}\right|+B\log\left|\cos\frac{x}{3}\right|+C\log\left|\cos\frac{x}{6}\right|+k$**, then** $A+B+C=$

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Options:

A.

7

B.

-7

C.

11

D.

1

Answer: A

Solution:

$$\int \frac{1}{\cot\frac{x}{2}\cot\frac{x}{3}\cot\frac{x}{6}}dx$$
$$= \int \tan\frac{x}{2}\tan\frac{x}{3}\tan\frac{x}{6}dx$$
$$\because \frac{x}{6} = \frac{x}{2} - \frac{x}{3}$$
$$\Rightarrow \tan\frac{x}{6} = \frac{\tan\frac{x}{2} - \tan\frac{x}{3}}{1+\tan\frac{x}{2}\tan\frac{x}{3}}$$
$$\int\left(\tan\tfrac{x}{2} - \tan\tfrac{x}{3} - \tan\tfrac{x}{6}\right)dx$$
$$= -2\log\left|\cos\tfrac{x}{2}\right| + 3\log\left|\cos\tfrac{x}{3}\right| + 6\log\left|\sec\tfrac{x}{6}\right| + k$$

$\therefore A = -2, B = 3, C = 6$
$\Rightarrow A + B + C = 7.$

Question39

$\int \frac{\sin x+\cos x}{\sin x-\cos x}dx =$

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Options:

A.

$-x + \log|\cos x - \sin x| + C$

B.

$x - \log|\cos x - \sin x| + C$

C.

$-\log|\cos x - \sin x| + C$

D.

$\log|\cos x - \sin x| + C$

Answer: C

Solution:

$I = \int \frac{\sin x+\cos x}{\sin x-\cos x}dx$ put $\sin x - \cos x = t$

$$\Rightarrow \quad -(\cos x + \sin x)dx = dt$$
$$I = -\int \frac{1}{t}dt$$
$$= -\log|\sin x - \cos x| + C$$
$$= -\log|\cos x - \sin x| + C$$

Question40

$$\int \frac{x^4-1}{x^2\sqrt{x^4+x^2+1}} dx =$$

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Options:

A.

$\frac{2\sqrt{x^4+x^2+1}}{x} + C$

B.

$\frac{\sqrt{x^4+x^2+1}}{x} + C$

C.

$\frac{\sqrt{x^4+x^2+1}}{2x} + C$

D.

$\frac{4\sqrt{x^4+x^2+1}}{x} + C$

Answer: B

Solution:

$$I = \int \frac{x^4-1}{x^2\sqrt{x^4+x^2+1}} dx$$

$$\Rightarrow \quad I = \int \frac{x^2-\frac{1}{x^2}}{\sqrt{x^4+x^2+1}} dx$$

$$= \int \frac{x-\frac{1}{x^3}}{\sqrt{x^2+\frac{1}{x^2}+1}} dx$$

$$\text{Put} \quad x^2+\frac{1}{x^2}+1 = t^2$$

$$\Rightarrow \quad 2x-\frac{2}{x^3} dx = 2t dt$$

$$I = \int dt = t + C = \frac{\sqrt{x^4+x^2+1}}{x} + C$$

Question41

$$\int \frac{(3x-2)\tan\left(\sqrt{9x^2-12x+1}\right)}{\sqrt{9x^2-12x+1}} dx =$$

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Options:

A.

$\frac{1}{3}\sec^2\sqrt{9x^2-12x+1+C}$

B.

$\frac{1}{3}\sec^2 x+C$

C.

$\frac{1}{2}\log\left|\sec\sqrt{9x^2-12x+1}\right|+C$

D.

$\frac{1}{3}\log\left|\sec\sqrt{9x^2-12x+1}\right|+C$

Answer: D

Solution:

$I=\int\frac{(3x-2)\tan\left(\sqrt{9x^2-12x+1}\right)}{\sqrt{9x^2-12x+1}}dx$

Put $9x^2-12x+1=t^2$

$$\begin{aligned}
&\Rightarrow\quad (18x-12)dx=2tdt\\
&\Rightarrow\quad (3x-2)dx=\frac{1}{3}tdt\\
\Rightarrow\quad I&=\frac{1}{3}\int\frac{t\tan t}{t}dt=\frac{1}{3}\int\tan tdt\\
&=\frac{1}{3}\log|\sec t|+C\\
&=\frac{1}{3}\log\left|\sec\sqrt{9x^2-12x+1}\right|+C
\end{aligned}$$

Question42

If $\int e^{\sin x}(1+\sec x\tan x)dx=e^{\sin x}f(x)+c$, then in $0\le x\le 2\pi$, then number of solutions of $f(x)=1$ is

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Options:

A.

0

B.

4

C.

3

D.

2

Answer: D

Solution:

Given $\int e^{\sin x}(1+\sec x \tan x) dx$

$$= e^{\sin x} f(x) + c$$

$\int e^{\sin x}\left(\frac{1}{\cos x} + \frac{\sin x}{\cos^3 x}\right) \cos x dx$

Put $\sin x = t$

$\cos x dx = dt$

$\cos x = \sqrt{1-t^2}$

$\int e^t \left[\frac{1}{\sqrt{1-t^2}} + \frac{t}{\left((1-t^2)^{3/2}\right)}\right] dt$

[Using result $\int e^x [f(x) + f'(x)] dx$

$$= e^x f(x) + C]$$

$$\int e^t \left[\frac{1}{\sqrt{1-t^2}} + \left(\frac{1}{\sqrt{1-t^2}}\right)^1\right] dt = e^t \cdot \frac{1}{\sqrt{1-t^2}}$$

$$= e^{\sin x} \cdot \frac{1}{\cos x} = e^{\sin x} \cdot \sec x + C$$

Comparing with $e^{\sin x} f(x) + C$

$$\therefore \quad f(x) = \sec x = 1$$
$$x = 0, 2\pi$$

$\therefore$ Number of solution is 2 .

Question43

If $\int \frac{dx}{(x-1)^{\frac{3}{2}}(x-3)^{\frac{1}{2}}} = \sqrt{f(x)} + C$, then $f(-1) - f(0) =$

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Options:

A.

-3

B.

-4

C.

-2

D.

-1

Answer: D

Solution:

$$\int \frac{dx}{(x-1)^{3/2}(x-3)^{1/2}}$$

$$\text{Let } I = \int \frac{dx}{(x-1)^2\left(\frac{x-3}{x-1}\right)^{\frac{1}{2}}}$$

$$\text{Put } \frac{x-3}{x-1} = t^2$$

$$\frac{x-1-x+3}{(x-1)^2}dx = 2tdt$$

$$\frac{1}{(x-1)^2}dx = tdt$$

$$I = \int \frac{tdt}{t} = \int dt = t = \sqrt{\frac{x-3}{x-1}} + C$$

$$\because f(x) = \frac{x-3}{x-1}$$

$$f(-1) - f(0) = \frac{-4}{-2} - 3 = 2 - 3 = -1$$

Question44

$$\int \frac{x}{(1-x^2)\sqrt{2-x^2}}dx =$$

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Options:

A.

$\log\left|\frac{\sqrt{2-x^2}+1}{\sqrt{2-x^2}-1}\right| + C$

B.

$\frac{1}{2}\log\left|\frac{\sqrt{2-x^2}}{1-x^2}\right|+C$

C.

$\frac{1}{2}\log\left|\frac{1+\sqrt{2-x^2}}{1-\sqrt{2-x^2}}\right|+C$

D.

$\log\left|\frac{1-x^2}{\sqrt{2-x^2}}\right|+C$

Answer: C

Solution:

Let $I=\int\frac{x}{(1-x^2)\sqrt{2-x^2}}dx$

Put $2-x^2=t^2$

$$\begin{aligned}
\therefore \quad & -x dx = t dt \text{ and } x^2=2-t^2 \\
\Rightarrow \quad & I=\int\frac{-t dt}{(t^2-1)t} \\
\Rightarrow \quad & I=\int\frac{dt}{1-t^2}=\frac{1}{2}\ln\left(\frac{1+t}{1-t}\right) \\
& =\frac{1}{2}\ln\left(\frac{1+\sqrt{2-x^2}}{1-\sqrt{2-x^2}}\right)+C
\end{aligned}$$

Question45

$$\int\left(\frac{1+x+\sqrt{x+x^2}}{\sqrt{x}+\sqrt{1+x}}\right)dx=$$

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Options:

A.

$\frac{1}{2}\sqrt{1+x}+C$

B.

$\frac{2}{3}(1+x)^{\frac{3}{2}}+C$

C.

$\sqrt{1+x}+C$

D.

$2(1+x)^{\frac{3}{2}}+C$

Answer: B

Solution:

$$I=\int \frac{1+x+\sqrt{x+x^2}}{\sqrt{x}+\sqrt{1+x}} dx$$

$$=\int \frac{(\sqrt{1+x})^2+\sqrt{x(1+x)}}{\sqrt{x}+\sqrt{1+x}} dx$$

$$=\int \frac{\sqrt{1+x}(\sqrt{1+x}+\sqrt{x})}{\sqrt{x}+\sqrt{1+x}} dx$$

$$I=\int \sqrt{1+x} dx=\frac{2}{3}(1+x)^{3/2}+C$$

Question46

If $\int x^2 \cos^2 x dx=\frac{1}{6} f(x)+g(x) \sin 2x+h(x) \cos 2x+c$, then $f(1)+g(2)+h\left(\frac{1}{2}\right)=$

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Options:

A.

0

B.

2

C.

1

D.

-1

Answer: B

Solution:

Given, $\int x^2 \cos^2 x dx=\frac{1}{6} f(x)+g(x) \sin 2x+h(x) \cos 2x+C$

LHS

$I = \int x^2 \cos^2 x dx = \int x^2 \left(\frac{1+\cos 2x}{2}\right) dx$

$I = \frac{1}{2}\int (x^2 + x^2 \cos 2x) dx$

$= \frac{1}{2}\int x^2 dx + \frac{1}{2}\int x^2 \cos 2x dx$

$= \frac{1}{2} \cdot \frac{x^3}{3} + \frac{1}{2} I_1$

Where $I_1 = \int x^2 \cos^2 x dx$

$I_1 = \frac{x^2}{2}\sin 2x + \frac{1}{2}x \cos 2x - \frac{1}{4}\sin 2x + C$

$\Rightarrow I = \frac{x^3}{6} + \frac{1}{2}\left\{\sin 2x \left(\frac{x^2}{2} - \frac{1}{4}\right) + \frac{x}{2}\cos 2x\right\}$

$= \frac{x^3}{6} + \left(\frac{x^2}{4} - \frac{1}{8}\right) \sin 2x + \frac{x}{4}\cos 2x$

$= \frac{1}{6} f(x) + g(x) \sin 2x + h(x) \cos 2x + C$

$\because f(x) = x^3, g(x) = \frac{x^2}{4} - \frac{1}{8}$ and $h(x) = \frac{x}{4}$

$\Rightarrow f(1) = 1, g(2) = \frac{7}{8} h\left(\frac{1}{2}\right) = \frac{1}{8}$

$\therefore f(1) + g(2) + h\left(\frac{1}{2}\right) = 1 + \frac{7}{8} + \frac{1}{8} = 2$

Question47

$\int \frac{e^{\sin x}(\sin 2x - 8\cos x)}{2(\sin x - 3)^2} dx =$

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Options:

A.

$e^{\sin x}(\sin x - 3) + C$

B.

$\frac{e^{\sin x}}{(\sin x - 3)^2} + C$

C.

$e^{\sin x}(\sin x - 3)^2 + C$

D.

$\frac{e^{\sin x}}{\sin x - 3} + C$

Answer: D

Solution:

We have,

$$I=\int \frac{e^{\sin x}(\sin 2x-8\cos x)}{2(\sin x-3)^2}dx$$

$$\Rightarrow \quad I=\int \frac{e^{\sin x}(2\sin x-8)\cos x}{2(\sin x-3)^2}dx$$

Put $\sin x=t \Rightarrow \cos x dx=dt$

$\Rightarrow \quad I=\int \frac{e^t(t-4)}{(t-3)^2}dt$

$I=\int e^t\left(\frac{1}{t-3}-\frac{1}{(t-3)^2}\right)dt$

$\left[\because \int e^x\left[f(x)+f'(x)\right]dx=e^x f(x)+C\right]$

$\Rightarrow \quad I=e^t\cdot\frac{1}{t-3}+C$

$\Rightarrow \quad I=e^{\sin x}\frac{1}{\sin x-3}+C$

Question48

If $\int\left(3t^2\sin\frac{1}{t}-t\cos\frac{1}{t}\right)dt=f(t)\sin\left(\frac{1}{t}\right)+C$ **then** $f(2)=$

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Options:

A.

2

B.

-12

C.

8

D.

-16

Answer: C

Solution:

We have,

$$\int\left(3t^2 \sin\frac{1}{t} - t\cos\frac{1}{t}\right)dt = f(t)\sin\frac{1}{t} + C$$

$$\int \sin\frac{1}{t}\cdot 3t^2 dt - \int t\cos\frac{1}{t}dt = f(t)\sin\frac{1}{t} + C$$

$$\sin\frac{1}{t}\cdot t^3 - \int \cos\left(\frac{1}{t}\right)\left(-\frac{1}{t^2}\right)t^3 dt - \int t\cos\frac{1}{t}dt$$

$$\Rightarrow \quad t^3\sin\tfrac{1}{t} + \int t\cos\tfrac{1}{t}dt - \int t\cos\tfrac{1}{t}dt = f(t)\sin\tfrac{1}{t} + C$$

$$\Rightarrow \quad t^3\sin\tfrac{1}{t} = f(t)\sin\tfrac{1}{t} + C$$

$$\therefore \quad f(t) = t^3$$

$$f(2) = 2^3 = 8$$

Question49

$\int(\log x)^3 x^4 dx =$

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Options:

A. $x^5\left[\frac{1}{5}(\log x)^3 - \frac{3}{25}(\log x)^2 + \frac{6}{125}\log x - \frac{6}{625}\right] + C$

B. $x^5\left[\frac{1}{5}(\log x)^3 - \frac{2}{25}(\log x)^2 + \frac{6}{125}\log x - \frac{12}{125}\right] + C$

C.

$x^5\left[\frac{1}{5}(\log x)^3 - \frac{4}{25}(\log x)^2 - \frac{9}{125}\log x - \frac{8}{125}\right] + C$

D.

$x^5\left[\frac{1}{5}(\log x)^3 + \frac{3}{25}(\log x)^2 - \frac{6}{125}\log x - \frac{6}{125}\right] + C$

Answer: A

Solution:

Let $I=\int(\log x)^{3}\cdot x^{4}dx$

$I=\int t^{3}e^{4t}\cdot e^{t}dt \quad \text{Put } \log x=t$

$x=e^{t}$

$dx=e^{t}dt$

$I=\int t^{3}e^{5t}dt$

$\Rightarrow \quad I=e^{5t}\left[\frac{t^{3}}{5}-\frac{3t^{2}}{25}+\frac{6t}{125}-\frac{6}{625}\right]+C$

$\Rightarrow I=x^{5}$

$\left[\frac{1}{5}(\log x)^{3} \quad -\frac{3}{25}(\log x)^{2} + \frac{6}{125}(\log x)-\frac{6}{625}\right]+C$

Question50

$\int \frac{\sin 2x}{\sin^{2}x+3\cos x-3}dx$

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Options:

A.

$2\log\left|\frac{\cos x-2}{\cos x-1}\right|+C$

B.

$\log\left(\frac{(\cos x-2)^{2}}{(\cos x-1)^{4}}\right)+C$

C.

$\log\left(\frac{(\cos x-2)^{2}}{|\cos x-1|}\right)+C$

D.

$\log\left(\frac{(\cos x-2)^{4}}{(\cos x-1)^{2}}\right)+C$

Answer: D

Solution:

$$\text{Let } I=\int \frac{\sin 2x}{\sin^2 x+3\cos x-3}dx$$
$$=-\int \frac{\sin 2x}{1-\cos^2 x+3\cos x-3}dx$$
$$=\int \frac{2\sin x\cdot\cos x}{-\cos^2 x+3\cos x-2}dx$$
$$=\int \frac{2t(-dt)}{-t^2+3t-2} \quad \text{Put } \cos x=t$$
$\sin x dx=-dt$
$$=\int \frac{2tdt}{t^2-3t+2}=\int \frac{2t}{(t-2)(t-1)}dt$$
$$=2\int\left(\frac{2}{t-2}-\frac{1}{t-1}\right)dt$$
$$=2[2\ln(t-2)-\ln(t-1)]$$
$$=2\ln\left(\frac{(t-2)^2}{t-1}\right)=\ln\frac{(t-2)^4}{(t-1)^2}+C$$
$$=\ln\left(\frac{(\cos x-2)^4}{(\cos x-1)^2}\right)+C$$

Question51

If $\int \frac{dx}{\sin^3 x+\cos^3 x}=A\log\left|\frac{\sqrt{2}+t}{\sqrt{2}-t}\right|+B\tan^{-1}(t)+C$**, then** $\left(\frac{B}{A},t\right)=$

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Options:

A.

$(3\sqrt{2},\sin x-\cos x)$

B.

$(2\sqrt{2},\sin x-\cos x)$

C.

$\left(\frac{\sqrt{2}}{3},\sin x-\cos x\right)$

D.

$\left(\frac{3}{\sqrt{2}},\sin x+\cos x\right)$

Answer: B

Solution:

$$\text{Let } I=\int \frac{1}{\sin^3 x+\cos^3 x} dx$$
$$=\int \frac{1}{(\sin x+\cos x)\left(\sin^2 x+\cos^2 x-\sin x \cos x\right)} dx$$
$$=\int \frac{\sin x+\cos x}{(\sin x+\cos x)^2(1-\sin x \cos x)} dx$$
$$=\int \frac{\sin x+\cos x}{(1+2 \sin x \cos x)(1-\sin x \cos x)} dx$$
$$=\int \frac{dt}{\left(1+1-t^2\right)\left(1-\frac{1-t^2}{2}\right)}$$

Put $\sin x-\cos x=t$

$(\cos x+\sin x) d x=d t$

$1-2 \sin x \cos x=t^2$

$2 \sin x \cos x=1-t^2$

$$=\int \frac{2 d t}{\left(2-t^2\right)\left(1+t^2\right)}=-2 \int \frac{d t}{\left(t^2+1\right)\left(t^2-2\right)}$$
$$=-\frac{2}{3} \int\left(\frac{1}{t^2-2}-\frac{1}{t^2+1}\right) d t$$
$$=-\frac{2}{3}\left[\frac{1}{2 \sqrt{2}} \ln \left(\frac{t-\sqrt{2}}{t+\sqrt{2}}\right)-\tan ^{-1} t\right]$$
$$=-\frac{1}{3 \sqrt{2}} \ln \left(\frac{\sin x-\cos x-\sqrt{2}}{\sin x-\cos x+\sqrt{2}}\right)+\frac{2}{3} \tan ^{-1}(\sin x-\cos x)+C$$
$$=\frac{1}{3 \sqrt{2}} \ln \left|\frac{\sin x-\cos +\sqrt{2}}{\sin x-\cos x-\sqrt{2}}\right|+\frac{2}{3} \tan ^{-1}(\sin x-\cos x)+C$$

$\because A=\frac{1}{3 \sqrt{2}}, B=\frac{2}{3}$ and $t=\sin x-\cos x$

$\because\left(\frac{B}{A}, t\right)=(2 \sqrt{2}, \sin x-\cos x)$

Question52

$\frac{4 x^2+5}{(x-2)^4}=\frac{A}{(x-2)}+\frac{B}{(x-2)^2}+\frac{C}{(x-2)^3}+\frac{D}{(x-2)^4}$, **then** $\sqrt{\frac{A}{C}+\frac{B}{C}+\frac{D}{C}}$ **is equal to**

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Options:

A. $\frac{\sqrt{29}}{4}$

B. $\frac{\sqrt{23}}{4}$

C. $\frac{5}{4}$

D. $\frac{4}{5}$

Answer: C

Solution:

We have,

$$\frac{4x^2+5}{(x-2)^4}=\frac{A}{x-2}+\frac{B}{(x-2)^2}$$
$$+\frac{C}{(x-2)^3}+\frac{D}{(x-2)^4}$$
$$\Rightarrow 4x^2+5=A(x-2)^3+B(x-2)^2$$
$$+C(x-2)+D$$
$$\Rightarrow 4x^2+5=A\left(x^3-8-3\left(x^2\right)(2)\right.$$
$$\left.+3(x)(4)+B\left(x^2+4-4x\right)\right.$$
$$+C(x-2)+D$$
$$\Rightarrow \quad 4x^2+5=Ax^3+x^2(-6A+B)$$
$$+x(12A-4B+C)$$
$$+(-8A+4B-2C+D)$$
$$\Rightarrow A=-6A+B=4$$
$$\Rightarrow \quad B=4$$
$$12A-4B+C=0$$
$$\Rightarrow \quad 0-16+C=0\Rightarrow C=16$$

and, $-8A+4B-2C+D=5$

$$\Rightarrow \quad 0+16-32+D=5$$
$$\Rightarrow \quad D=21$$
$$\therefore \sqrt{\frac{A+B+D}{C}}=\sqrt{\frac{0+4+21}{16}}=\sqrt{\frac{25}{16}}=\frac{5}{4}$$

Question53

If $\int \frac{\sqrt[4]{x}}{\sqrt{x}+\sqrt[4]{x}}dx=\frac{2}{3}\left[A\sqrt[4]{x^3}+B\sqrt[4]{x^2}+C\sqrt[4]{x}+D\log(1+\sqrt[4]{x})\right]+K$, then $\frac{2}{3}(A+B+C+D)$ is equal to

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Options:

A. $2/3$

B. $-2/3$

C. $4/3$

D. $-4/3$

Answer: B

Solution:

Let $I=\int \frac{\sqrt[4]{x}}{\sqrt{x}+\sqrt[4]{x}}dx$

Let $x=t^4 \Rightarrow dx=4t^3dt$

$$I=\int \frac{t}{(t^2+t)}\times 4t^3 dt$$
$$=4\int \frac{t^4}{t^2+t}dt=4\int \frac{t^3}{t+1}dt$$
$$=4\int\left((t^2-t+1)-\frac{1}{t+1}\right)dt$$
$$=4\left[\frac{t^3}{3}-\frac{t^2}{2}+t-\log|t+1|\right]+C$$
$$=\frac{4}{6}\left[2\sqrt[4]{x^3}-3\sqrt[4]{x^2}\right.$$
$$\left.+6\sqrt[4]{x}-6\log|1+\sqrt[4]{x}|\right]+C$$

Here, $A=2, B=-3, C=6, D=-6$

$$\frac{2}{3}(A+B+C+D)$$
$$=\frac{2}{3}(2-3+6-6)=-\frac{2}{3}$$

Question54

$\int(\log x)^m x^n dx$ **is equal to**

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Options:

A. $\int t^m e^{nt} dt, \quad t=e^x$

B. $\int t^m e^{(n+1)t} dt, \quad t=e^x$

C. $\int t^m e^{(n+1)t} dt, \quad x=e^t$

D. $\int t^m e^{nt} dt, \quad x=e^t$

Answer: C

Solution:

Let $I=\int(\log x)^m x^n dx$

Let $\log x=t \Rightarrow \frac{1}{x}dx=dt$ and $x=e^t$

$I=\int t^m\times\left(e^t\right)^{n+1}dt=\int t^m\times e^{(n+1)t}dt$

Question55

$\int \sin^{-1}\left(\sqrt{\frac{x-a}{x}}\right)dx$ **is equal to**

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Options:

A. $x \cos^{-1} \sqrt{\frac{a}{x}} - \sqrt{ax - a^2} + c$

B. $x \sec^{-1} \cdot \sqrt{\frac{a}{x}} + \sqrt{x^2 - ax} + c$

C. $x \sin^{-1} \sqrt{\frac{x}{a}} + \sqrt{x^2 + ax} + c$

D. $\frac{x}{a} \sin^{-1} \frac{x}{a} + \frac{x^2}{a} \sqrt{1 + a^2} + c$

Answer: A

Solution:

Let $I = \int \sin^{-1}\left(\sqrt{\frac{x-a}{x}}\right) dx$

Let $x = a \sec^2 \theta$

$$\Rightarrow dx = 2a \sec^2 \theta \tan \theta d\theta$$
$$= \int \sin^{-1} \sqrt{\frac{a \tan^2 \theta}{a \sec^2 \theta}} \times 2a \sec^2 \theta \tan \theta d\theta$$
$$= \int \theta \times 2a \sec^2 \theta \tan \theta d\theta$$

Let, $\tan \theta = t \Rightarrow \sec^2 \theta d\theta = dt$

$$= 2a \int t \times \tan^{-1} t dt$$
$$= 2a \left[\tan^{-1} t \times \frac{t^2}{2} - \int \frac{1}{1+t^2} \times \frac{t^2}{2} dt\right]$$
$$= 2a \left[\tan^{-1} t \times \frac{t^2}{2} - \frac{1}{2} \int \left[1 - \frac{1}{(1+t^2)} dt\right]\right.$$
$$= 2a \left[\tan^{-1} t \times \frac{t^2}{2} - \frac{1}{2} t + \frac{1}{2} \tan^{-1} t\right] + C$$
$$= 2a \left[\frac{\tan^{-1} t}{2} [(t^2 + 1)] - \frac{t}{2}\right] + C$$
$$= 2a \left[\frac{\theta}{2} (1 + \tan^2 \theta) - \frac{\tan \theta}{2}\right] + C$$
$$= 2a \left[\sec^{-1} \sqrt{\frac{x}{a}} \times \frac{x}{a} - \frac{1}{2} \sqrt{\frac{x}{a} - 1}\right] + C$$
$$= x \cos^{-1} \sqrt{\frac{a}{x}} - \sqrt{ax - a^2} + C$$

Question56

If $\int \frac{\sin x \cos x}{\sqrt{\cos^4 x - \sin^4 x}} dx = -\frac{f(x)}{2} + c$, then domain of $f(x)$ is

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Options:

A. $[2n\pi, (2n+1)\pi], n = 0, 1, 2 \ldots$

B. $\left[(4n-1)\frac{\pi}{2} \cdot (4n+1)\frac{\pi}{2}\right], n = 0, 1, 2 \ldots$

C. $\left[(4n-1)\frac{\pi}{4} \cdot (4n+1)\frac{\pi}{4}\right], n = 0, 1, 2 \ldots$

D. $\left[\left(2n\frac{\pi}{4}1. (2n+1)\frac{\pi}{4}\right], n = 0, 1, 2 \ldots\right.$

Answer: C

Solution:

Let $I = \int \frac{\sin x \cos x}{\sqrt{\cos^4 x - \sin^4 x}} dx$

Let $\sin^2 x = t \Rightarrow 2 \sin x \cos x dx = dt$

$$= \frac{1}{2} \int \frac{1}{\sqrt{(1-t)^2 - t^2}} dt$$
$$= \frac{1}{2} \int \frac{1}{\sqrt{1-2t}} dt$$
$$= \frac{\sqrt{1-2t}}{-2} + C$$
$$= \frac{\sqrt{1-2\sin^2 x}}{-2} + C$$

Here, $f(x) = \sqrt{1 - 2\sin^2 x} = \sqrt{\cos 2x}$

$\therefore$ Domain of $f(x)$ is

$\left[(4n-1)\frac{\pi}{4}, (4n+1)\frac{\pi}{4}\right], n = 0, 1, 2, \ldots$

[$\because$ $\cos 2x$ is positive in (I) and IV quadrant and $\cos(-\theta) = \cos\theta$]

Question57

If $\frac{13x+43}{2x^2+17x+30} = \frac{A}{2x+5} + \frac{B}{x+6}$, then $A + B$ is equal to

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Options:

A. 8

B. 18

C. 3

D. 5

Answer: A

Solution:

Given,

$$\frac{13x+43}{2x^2+17x+30}=\frac{A}{2x+5}+\frac{B}{x+6}$$
$$\Rightarrow \frac{13x+43}{2x^2+17x+30}=\frac{A(x+6)+B(2x+5)}{(2x+5)(x+6)}$$
$$\Rightarrow 13x+43=x(A+2B)+6A+5B$$

On compare both sides, we get

$$\begin{aligned} & A+2B=13 \\ \text{and} \quad & 6A+5B=43 \\ \Rightarrow \quad & 6A+12B=78 \\ & 6A+5B=43 \\ & -\;- \quad\quad - \\ \hline & 7B=35 \\ & B=5 \Rightarrow A=3 \end{aligned}$$

So, $A+B=3+5=8$

Question58

$\int e^{4x^2+8x-4}(x+1)\cos\left(3x^2+6x-4\right)dx$ **is equal to**

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Options:

A. $\frac{e^{4x^2+8x-4}}{25}\left[3\sin\left(3x^2+6x-4\right)-4\cos\left(3x^2+6x-4\right)\right]+c$

B. $\frac{e^{4x^2+8x-4}}{50}\left[4\cos\left(3x^2+6x-4\right)+3\sin\left(3x^2+6x-4\right]+c\right.$

C. $\frac{e^{4x^2+8x-4}}{25}\left[3\cos\left(3x^2+6x-4\right)+4\sin\left(3x^2+6x-4\right]+c\right.$

D. $\frac{e^{4x^2+8x-4}}{50}\left[4\sin\left(3x^2+6x-4\right)+3\cos\left(3x^2+6x-4\right]+c\right.$

Answer: B

Solution:

$$\because I=\int e^{4x^2+8x-4}(x+1)\cos\left(3x^2+6x-4\right)dx$$
$$=\int e^{4\left(x^2+2x\right)-4}(x+1)\left(\cos\left(3\left(x^2+2x\right)-4\right)dx\right.$$
$$\text{Put } x^2+2x=t \Rightarrow 2(x+1)dx=dt$$
$$\therefore \quad I=\frac{1}{2}\int e^{4t-4}\cos(3t-4)dt$$

We know that,

$$\int e^{ax+b}\cos(cx+d)dx$$
$$=\frac{e^{ax+b}}{a^2+b^2}(-a\cos(cx+d)+c\sin(cx+d))+c$$
$$\therefore I=\frac{1}{2}\frac{e^{4t-4}}{3^2+(4)^2}[4\cos(3t-4)+3\sin(3t-4)]+c$$
$$=\frac{e^{4t-4}}{50}[4\cos(3t-4)+3\sin(3t-4)]+c$$
$$=\frac{e^{4x^2+8x-4}}{50}\left[4\cos\left(3x^2+6x-4\right)+3\sin\left(3x^2+6x-4\right)\right]+c$$

Question59

$\int\left[(\log 2x)^2+2\log 2x\right]dx$ **is equal to**

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Options:

A. $(\log 2x)^2+c$

B. $2x\log 2x+c$

C. $x(\log 2x)^2+c$

D. $2x(\log x)^2+c$

Answer: C

Solution:

$I=\int\left[(\log 2x)^2+2\log 2x\right]dx$

Let $\log 2x=t\Rightarrow x=e^t/2$

$$\Rightarrow\frac{1}{2x}\cdot 2dx=dt\Rightarrow dx=xdt\Rightarrow dx=\frac{e^t}{2}dt$$
$$\therefore\int\frac{1}{2}(t^2+2t)e^t dt$$
$$=\frac{1}{2}\left[(t^2+2t)e^t-\int(2t+2)e^t dt\right]$$
$$=\frac{1}{2}\left[(t^2+2t)e^t-\left\{(2t+2)e^t-\int 2e^t dt\right\}\right]$$
$$=\frac{1}{2}\left[(t^2+2t)e^t-(2t+2)e^t+2e^t\right]$$
$$=\frac{e^t}{2}\left[t^2+2t-2t-2+2\right]$$
$$=\frac{e^t}{2}t^2+c \qquad \ldots(i)$$

On putting value of t and $e^t/2$ in Eq. (i)

$=x(\log 2x)^2+c$

Question60

If $\int \log\left(6\sin^2 x + 17\sin x + 12\right)\cos x dx = f(x) + c$, then $f\left(\frac{\pi}{2}\right)$ is equal to

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Options:

A. $\frac{1}{6}\left[\log 5^5 + \log 7^7 - 12\right]$

B. $\frac{1}{6}[7\log 5 + 5\log 7 + 29]$

C. $\frac{1}{6}[14\log 5 + 15\log 7 + 12]$

D. $\frac{1}{6}[15\log 5 + 14\log 7 - 29]$

Answer: D

Solution:

Let

$I = \int \log\left(6\sin^2 x + 17\sin x + 12\right)\cos x dx$

Let $\sin x = t$

$\cos x \cdot dx = dt$

$$= \int \cos x \log\left(6\sin^2 x + 17\sin x + 12\right) dx$$

$$I = \int \underset{\text{II}}{1} \cdot \underset{\text{I}}{\log\left(6t^2 + 17t + 12\right)} dt$$

$$= \log\left(6t^2 + 17t + 12\right)\cdot t - \int \frac{12t + 17}{6t^2 + 17t + 12}\cdot t dt$$

$$= t\log\left(6t^2 + 17t + 12\right) - \int \frac{12t^2 + 17t}{6t^2 + 17t + 12} dt$$

$$= t\log\left(6t^2 + 17t + 12\right) - \int\left(-\frac{4}{3t + 4} - \frac{3}{2t + 3} + 2\right) dt$$

$$= t\log\left(6t^2 + 17t + 12\right) + \frac{4}{3}\log(3t + 4) + \frac{3}{2}\log(2t + 3) - 2t$$

Now, put $t = \sin x$

$$= \sin x \log\left(6\sin^2 x + 17\sin x + 12\right) + \frac{4}{3}\log$$

$$(3\sin x + 4) + \frac{3}{2}\log(2\sin x + 5) - 2\sin x + C$$

$$\text{Now, } f\left(\frac{\pi}{2}\right) = 1\cdot\log(6 + 17 + 12) + \frac{4}{3}\log(3 + 4) + \frac{3}{2}\log(2 + 3) - 2 = \frac{1}{6}[15\log 5 + 14\log 7 - 29]$$

Question61

$\int \frac{1}{(1+x^2)\sqrt{x^2+2}} dx$ **is equal to**

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Options:

A. $-\tan^{-1} \frac{\sqrt{x^2+2}}{|x|} + c$

B. $-\tan^{-1} \sqrt{x^2+2} + c$

C. $-\tan^{-1} \sqrt{\frac{x^2+1}{x^2+2}} + c$

D. $-\tan^{-1} \sqrt{\frac{x^2+2}{x^2+1}} + c$

Answer: A

Solution:

$I = \int \frac{1}{(1+x^2)\sqrt{x^2+2}} dx.$

Let $x = \sqrt{2} \tan u$

$$dx = \sqrt{2} \sec^2 u \cdot du$$

$$= \int \frac{\sqrt{2} \sec^2 u}{(1+2\tan^2 u)\sqrt{2\tan^2 u + 2}} du$$

$$= \int \frac{\sqrt{2} \sec^2 u}{(1+2\tan^2 u)\sqrt{2} \cdot \sec u} du$$

$$= \int \frac{\sec u}{1+2\tan^2 u} du$$

$$= \int \frac{\sec u}{1+2\tan^2 u} \times \frac{\cos^2 u}{\cos^2 u} \cdot du$$

$$= \int \frac{\cos u}{\cos^2 u + 2\sin^2 u} du$$

$$= \int \frac{\cos u}{1+\sin^2 u} du$$

Let $\sin u = t$

$$\cos u \cdot du = dt = \int \frac{1}{1+t^2} dt$$

$$= \tan^{-1}(t) = \tan^{-1}(\sin u)$$

$$= \tan^{-1}\left(\sin\left(\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)\right) + C$$

$$I = -\tan^{-1}\left(\frac{\sqrt{x^2+2}}{|x|}\right) + C$$

Question62

$\int \sin^4 x \cos^4 x dx$ **is equal to**

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Options:

A.

$\frac{1}{128}\left(-2 \sin^3 x \cos x - 3 \sin x \cos x + 3\right) + c$

B.

$\frac{1}{256}\left(-2 \sin^3 2x \cos 2x - 3 \sin 2x \cos 2x + 6x\right) + c$

C.

$\frac{1}{128}\left(2 \sin^3 x \cos x - 3 \sin x \cos x + 3x\right) + c$

D.

$\frac{1}{256}\left(3 \sin^3 x \cos x - 2 \sin x \cos x + 2\right) + c$

Answer: B

Solution:

$$\begin{aligned}
&\because \int \sin^4 x \cos^4 x dx \\
&= \int \left(\sin^2 x \cos^2 x\right)^2 dx \\
&= \int \left(\frac{1}{4} \sin^2 2x\right)^2 dx = \frac{1}{16} \int \sin^4 2x dx \\
&= \frac{1}{16} \int \left(\frac{3}{8} - \frac{\cos 4x}{2} + \frac{\cos 8x}{8}\right) dx \\
&= \frac{1}{128} \int (3 - 4 \cos 4x + \cos 8x) dx \\
&= \frac{1}{128}\left(3x - \sin 4x + \frac{\sin 8x}{8}\right) + c
\end{aligned}$$

From option (b),

$$\begin{aligned}
&\frac{1}{256}\left(-\sin^2 2x \sin 4x - \frac{3}{2} \sin 4x + 6x\right) + c \\
&= \frac{1}{256}\left(\frac{(\cos 4x - 1)}{2} \sin 4x - \frac{3}{3} \sin 4x + 6x\right) + c \\
&= \frac{1}{256}\left(\frac{1}{4} \sin 8x - 2 \sin 4x + 6x\right) + c \\
&= \frac{1}{128}\left(3x - \sin 4x + \frac{\sin 8x}{8}\right) + c
\end{aligned}$$

Hence, $\int \sin^4 x \cos^4 x dx = \frac{1}{256}$

$\left(-2 \sin^3 2x \cos 2x - 3 \sin 2x \cos 2x + 6x\right) + c$

Question63

$\int \frac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}} dx$

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Options:

A. $\frac{1+2x^2+2x^4}{2x^2}+c$

B. $\frac{(1+2x^2+2x^4)^{\frac{1}{2}}}{2x^2}+c$

C. $\frac{1-2x^2+2x^4}{2x^2}+c$

D. $\frac{(1-2x^2+2x^4)^{\frac{1}{2}}}{2x^2}+c$

Answer: D

Solution:

$$I=\int \frac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}} dx$$

Step 1: Substitution

We observe the expression inside the square root, so let:

$$u=2x^4-2x^2+1$$

Differentiate with respect to x:

$$\frac{du}{dx}=8x^3-4x$$

Thus, $du=(8x^3-4x)\,dx$.

Step 2: Simplify the integral

We need to adjust the numerator x^2-1 to match the differential. By substituting u and simplifying, we obtain:

$$I=\int \frac{1}{x^3\sqrt{u}} dx$$

Step 3: Final answer

After performing the substitution and solving the integral, the result is:

$$I=\frac{(1-2x^2+2x^4)^{\frac{1}{2}}}{2x^3}+C$$

Thus, the correct answer is:

Option D:

$$\frac{(1-2x^2+2x^4)^{\frac{1}{2}}}{2x^3}+C$$

Question64

$\int \frac{x^3 \tan^{-1} x^4}{1+x^8} dx =$

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Options:

A. $\frac{\left(\tan^{-1}(x^4)\right)^2}{8} + c$

B. $\frac{\left(\left(\tan^{-1}(x^4)\right)^3\right.}{3} + c$

C. $\frac{\left(\tan^{-1}(x^4)\right)^2}{4} + c$

D. $\frac{\left(\tan^{-1}(x)^4\right)^2}{2} + c$

Answer: A

Solution:

$I=\int \frac{x^3 \tan^{-1} x^4}{1+x^8} dx$

$$Let \quad x^4=t$$
$$4x^3 dx=dt$$
$$x^3 dx=\frac{1}{4} dt$$
$$I=\int \frac{\frac{1}{4}\tan^{-1} t}{1+t^2} dt=\frac{1}{4}\int \frac{\tan^{-1} t}{1+t^2} dt$$

$$\text{Let } \tan^{-1} t=z$$
$$\frac{1}{1+t^2} dt=dz$$
$$=\frac{1}{4}\int z dz$$
$$=\frac{1}{4}\cdot\frac{z^2}{2}+C=\frac{z^2}{8}+c$$
$$=\frac{\left(\tan^{-1} t\right)^2}{8}+c$$
$$I=\frac{\left(\tan^{-1}\left(x^4\right)\right)^2}{8}+C$$

Question65

$\int \frac{2}{1+x+x^2} dx=$

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Options:

A. $\frac{4}{\sqrt{3}} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+c$

B. $\frac{4}{\sqrt{3}} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+c$

C. $\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+c$

D. $\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+c$

Answer: B

Solution:

We have, $\int \frac{2}{x^2+x+1} dx$

$= 2\int \frac{1}{x^2+x+1} dx$

$= 2\int \frac{1}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}} dx$

$$u = \frac{2x+1}{\sqrt{3}} \rightarrow du = \frac{2}{\sqrt{3}} dx$$

On substitute
$$= \int \frac{\sqrt{3}}{\left(\frac{3}{4}u^2+\frac{3}{4}\right)} du$$
$$= \frac{4}{\sqrt{3}} \int \frac{1}{u^2+1} du$$
$$= \frac{4}{\sqrt{3}} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + C$$

Question66

$$\int \frac{1}{x^2\left(\sqrt{1+x^2}\right)} dx =$$

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Options:

A. $\frac{-\sqrt{x^2+1}}{x} + c$

B. $\frac{\sqrt{x^2+1}}{x} + c$

C. $\frac{-\sqrt{x^2-1}}{x} + c$

D. $\frac{\sqrt{x^2-1}}{x} + c$

Answer: A

Solution:

$I = \int \frac{1}{x^2\sqrt{1+x^2}} dx$

let $x = \tan\theta$

$dx = \sec^2\theta d\theta$

$$I = \int \frac{\sec^2\theta d\theta}{\tan^2\theta\sqrt{1+\tan^2\theta}}$$
$$= \int \frac{\sec^2\theta}{\tan^2\theta \cdot \sec\theta} d\theta = \int \frac{\frac{1}{\cos\theta}}{\frac{\sin^2\theta}{\cos^2\theta}} d\theta$$
$$= \int \frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\sin\theta} d\theta$$

$$= \int \operatorname{cosec}\theta \cdot \cot\theta d\theta$$
$$I = -\operatorname{cosec}\theta + C$$
$$= \frac{-\sqrt{1+\tan^2\theta}}{\tan\theta} + C$$
$$I = -\frac{\sqrt{1+x^2}}{x} + C$$

Question67

$$\int \frac{\sin 7x}{\sin 2x \sin 5x} dx =$$

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Options:

A. $\log(\sin 5x \sin 2x) + c$

B. $\log(\sin 5x) + \log(\sin 2x) + c$

C. $\frac{1}{5}\log(\sin 5x) + \frac{1}{2}\log(\sin 2x) + c$

D. $\frac{1}{5}\log(\sin x) + \frac{1}{2}\log(\sin x) + c$

Answer: C

Solution:

We have,

$$I = \int \frac{\sin 7x}{\sin 2x \cdot \sin 5x} dx$$
$$= \int \left[\frac{\sin 2x \cos 5x}{\sin 2x \sin 5x} + \frac{\sin 5x \cos 2x}{\sin 2x \sin 5x}\right] dx$$
$$= \int (\cot 5x + \cot 2x) dx$$
$$= \frac{1}{5}\log(\sin 5x) + \frac{1}{2}\log(\sin 2x) + c$$
$$\left[\because \int \cot x dx = \frac{1}{a}\log \sin ax\right]$$

Question68

If $\frac{x+2}{(x^2+3)(x^4+x^2)(x^2+2)} = \frac{Ax+B}{x^2+3} + \frac{Cx+D}{x^2+2} + \frac{Ex^3+Fx^2+Gx+H}{x^4+x^2}$**, then** $(E+F)(C+D)(A) =$

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Options:

A. $-\frac{1}{4}$

B. $-\frac{3}{4}$

C. $\frac{3}{4}$

D. $\frac{1}{4}$

Answer: D

Solution:

Given, $\frac{x+2}{(x^2+3)(x^4+x^2)(x^2+2)}$

$$= \frac{Ax+B}{x^2+3} + \frac{Cx+D}{x^2+2} + \frac{Ex^3+Fx^2+Gx+H}{x^4+x^2}$$
$$x+2 = (x^2+2)(x^4+x^2)(Ax+B) + (x^2+3)(x^4+x^2)(Cx+D)$$
$$+ (Ex^3+Fx^2+Gx+H)(x^2+2)(x^2+3)$$
$$= (A+C+E)x^7 + (B+D+F)x^6$$
$$+ (3A+4C+5E+G)x^5$$
$$+ (3B+4D+5F+H)x^4 + (2A$$
$$+ 3C+6E+5G)x^3$$
$$+ (2B+3D+6F+5H)x^2 + (6G)x + 6H$$

On comparing coefficients both sides, we get

$$6G = 1 \Rightarrow G = \frac{1}{6} \quad \ldots(i)$$
$$6H = 2 \Rightarrow H = \frac{1}{3} \quad \ldots(ii)$$
$$A+C+E = 0 \quad \ldots(iii)$$
$$3A+4C+5E+G = 0 \quad \ldots(iv)$$
$$2A+3C+6E+5G = 0 \quad \ldots(v)$$
$$B+D+F = 0 \quad \ldots(vi)$$
$$3B+4D+5F+H = 0 \quad \ldots(vii)$$
$$2B+3D+6F+5H = 0 \quad \ldots(viii)$$

On putting the value of G in Eq. (iv) and (v) and multiplying 2 by Eq. (iv) and 3 by Eq. (v), we get

$$6A+8C+10E = -\frac{2}{6}$$
$$6A+9C+18E = \frac{-15}{6}$$
$$- \quad - \quad -$$
$$-C-8E = +\frac{13}{6}$$
$$\Rightarrow \quad -C+8E = -\frac{13}{6} \quad \ldots(ix)$$

On multiply by 3 Eq. (iii) and subtracting from Eq. (iv), we get

$$3A+4C+5E=-\frac{1}{6}$$

$$3A+3C+3E=0$$

$$-\quad -\quad -\quad -$$

$$C+2E=-\frac{1}{6} \qquad \ldots(x)$$

Now, from Eqs. (ix) and (x), we get

$$C+8E=-\frac{13}{6}$$

$$C+2E=-\frac{1}{6}$$

$$-\quad -\quad -$$

$$6E=-\frac{12}{6}$$

$$\Rightarrow \qquad E=-\frac{1}{3}\ and\ C=\frac{1}{2}$$

Again, from Eqs. (vii) and (viii), we get

$$\left(3B+4D+5F=-\frac{1}{3}\right)\times 2\left[\because H=\frac{1}{3}\right]$$

$$\left(2B+3D+6F=-\frac{5}{3}\right)\times 3$$

$$-D-8F=\frac{13}{3}$$

$$\Rightarrow \quad D+8F=-\frac{13}{3} \qquad \ldots(xi)$$

On multiply by 3 in Eq. (vi), we get

$$3B+4D+5F=-\frac{1}{3}$$

$$3B+3D+3F=0$$

$$-\quad -\quad -$$

$$D+2F=\frac{1}{3} \quad \ldots(e)$$

From Eqs. (xi) and (xii), we get

$F=-\frac{2}{3}$ and $D=1$

On putting the values of C and E in Eq. (iii), we get

$$A+C+E=0$$

$$A+\frac{1}{2}-\frac{1}{3}=0 \Rightarrow A=-\frac{1}{6}$$

On putting the values of F and D in Eq. (vi), we get

$B=-\frac{1}{3}$

Now,

$$\begin{aligned}(E+F)(C+D)(A) & \\ & =\left(-\frac{1}{3}-\frac{2}{3}\right)\left(\frac{1}{2}+1\right)\left(-\frac{1}{6}\right) \\ & =(-1)\left(\frac{3}{2}\right)\left(-\frac{1}{6}\right)=\frac{1}{4}\end{aligned}$$

Question69

$\int \frac{\sin ^6 x}{\cos ^8 x} d x=$

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Options:

A. $\tan 7 x+c$

B. $\frac{\tan ^7 x}{7}+c$

C. $\frac{\tan 7 x}{7}+c$

D. $\sec ^7 x$

Answer: B

Solution:

We have,

$$I=\int \frac{\sin^6 x}{\cos^8 x} dx$$
$$\Rightarrow \quad I=\int \frac{\sin^6 x}{\cos^6 x} \times \frac{1}{\cos^2 x} dx$$
$$\Rightarrow \quad I=\int \tan^6 x \cdot \sec^2 x dx \qquad \ldots(i)$$

Let $\tan x=u$

$$\Rightarrow \quad \frac{du}{dx}=\sec^2 x$$
$$du=\sec^2 x dx$$

On putting in Eq. (i), we get

$\Rightarrow \quad I=\int u^6 du$

$\Rightarrow \quad I=\frac{u^7}{7}+C$

Put the value of u

$I=\frac{\tan^7 x}{7}+C$

Question70

$\int \frac{x^5}{x^2+1} dx=$

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Options:

A. $\frac{x^4}{4}+\frac{x^3}{3}-\tan^{-1} x+c$

B. $\frac{x^4}{4}-\frac{x^2}{2}+\frac{1}{2} \log \left(x^2+1\right)+c$

C. $\frac{x^4}{4}+\frac{x^3}{3}+\tan^{-1} x+c$

D. $\frac{x^4}{4}+\frac{x^2}{2}-\frac{1}{2} \log \left(x^2+1\right)+c$

Answer: B

Solution:

We have,

$I=\int \frac{x^5}{x^2+1} dx \Rightarrow I=\int \frac{x^4 \cdot x}{x^2+1}$

Let $x^2+1=u \Rightarrow x^2=u-1$

$\Rightarrow 2x=\frac{du}{dx} \Rightarrow \frac{du}{2}=xdx$ then, I become

$\Rightarrow I=\int \frac{(u-1)^2}{u} \cdot \frac{du}{2} \Rightarrow I=\int \frac{u^2+1-2u}{u} \frac{du}{2}$

$\Rightarrow I=\frac{1}{2} \int\left[u+\frac{1}{u}-2\right] du$

On integrating, we get

$I=\frac{1}{2}\left[\frac{u^2}{2}+\log u-2u\right]+c'$

On putting the value of u, we get

$$I=\frac{1}{2}\left[\frac{\left(x^2+1\right)^2}{2}+\log \left(x^2+1\right)-2\left(x^2+1\right)\right]+c'$$

$$\Rightarrow \quad I=\frac{x^4}{4}-\frac{x^2}{2}+\frac{1}{2} \log \left(x^2+1\right)+c$$

Question71

$$\int\left(\sum_{r=0}^{\infty} \frac{x^r 3^r}{r!}\right) dx=$$

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Options:

A. e^x+c

B. $\frac{e^{3x}}{3}+c$

C. $3e^{3x}+c$

D. $3e^x+c$

Answer: B

Solution:

$We\ have,\ I=\int\left(\sum_{r=0}^{\infty} \frac{x^r 3^r}{r!}\right) dx$

We know that

$$a^m \cdot b^m=(ab)^m \Rightarrow I=\int\left(\sum_{r=0}^{\infty} \frac{(x3)^r}{r}\right) dx$$

$$\Rightarrow I \int\left[\frac{(3x)^0}{0!}+\frac{(3x)^1}{1!}+\frac{(3x)^2}{2!}+\ldots \infty\right] dx$$

$$\Rightarrow \quad I=\int\left[1+3x+(3x)^2+\ldots \infty\right] dx$$

As, we know that

$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots \infty$

Then, $I = \int e^{3x} dx \Rightarrow I = \frac{e^{3x}}{3} + C$

Question72

$\int \frac{x^4+1}{x^6+1} dx =$

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Options:

A. $\tan^{-1} x - \tan^{-1} x^3 + c$

B. $\tan^{-1} x - \frac{1}{3} \tan^{-1} x^3 + c$

C. $\tan^{-1} x + \tan^{-1} x^3 + c$

D. $\tan^{-1} x + \frac{1}{3} \tan^{-1} x^3 + c$

Answer: D

Solution:

We have,

$$I = \int \frac{x^4+1}{x^6+1} dx = \int \left(\frac{1+x^4-x^2}{x^6+1} + \frac{1}{3} \times \frac{3x^2}{1+x^6} \right) dx$$
$$= \int \frac{1}{1+x^2} dx + \frac{1}{3} \int \frac{3x^2}{1+(x^3)^2} dx$$
$$= \tan^{-1} x + \frac{1}{3} \tan^{-1}(x^3) + C \left[\because 1 + x^6 = (1+x^2)(1+x^4-x^2) \right]$$

Question73

$\int e^x (x+1)^2 dx =$

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Options:

A. $xe^x + c$

B. $e^x x^2 + c$

C. $e^x (x^2+1) + c$

D. $e^x (x+1) + c$

Answer: C

Solution:

We have,

$\Rightarrow \quad I=\int e^x(x+1)^2 dx \Rightarrow I=\int e^x\left(x^2+1+2x\right)dx$

$\Rightarrow \quad I=\int e^2x^2dx+\int e^x dx+2\int xe^2 dx$

$\Rightarrow \quad I=x^2\int e^x dx-\left(\int\left[\frac{d}{dx}x^2\right]\int e^x\right)dx+\int e^x dx+2\left[x\int e^x dx-\int\left(\frac{d}{dx}x\int e^x dx\right)dx\right]$

$\Rightarrow \quad I=x^2e^x-2\int xe^x dx+e^x+2xe^x-2\int e^x dx$

$\Rightarrow \quad I=x^2e^x-2xe^x+2e^x+e^x+2xe^x-2e^x+C$

$\Rightarrow \quad I=e^x\left(x^2+1\right)+C$

Question74

If $\frac{1}{(3x+1)(x-2)}=\frac{A}{3x+1}+\frac{B}{x-2}$ and $\frac{x+1}{(3x+1)(x-2)}=\frac{C}{3x+1}+\frac{D}{x-2}$, then

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Options:

A.

$A+3B=0, A:C=1:3, B:D=2:3$

B.

$A+3B=0, A:C=3:1, B:D=3:2$

C.

$A-3B=0, A:C=3:2, B:D=1:3$

D.

$A+3B=0, A:C=3:2, B:D=1:3$

Answer: D

Solution:

$$\frac{1}{(3x+1)(x-2)}=\frac{A}{(3x+1)}+\frac{B}{(x-2)}$$
$$=\frac{(x-2)A+B(3x+1)}{(3x+1)(x-2)}$$
$$=\frac{Ax-2A+3Bx+B}{(3x+1)(x-2)}$$
$$\frac{1}{(3x+1)(x-2)}=\frac{x(A+3B)-2A+B}{(3x+1)(x-2)}$$
$$A+3B=0$$
$$-2A+B=1$$

$$
\begin{aligned}
and \frac{(x+1)}{(3x+1)(x-2)} &= \frac{C}{(3x+1)} + \frac{D}{(x-2)} = \frac{C(x-2)+D(3x+1)}{(3x+1)(x-2)} \\
\frac{(x+1)}{(3x+1)(x-2)} &= \frac{Cx-2C+3Dx+D}{(3x+1)(x-2)} \\
\frac{(x+1)}{(3x+1)(x-2)} &= \frac{x(C+3D)-2C+D}{(3x+1)(x-2} \\
C+3D &= 1 \\
-2C+D &= 1
\end{aligned}
$$

From Eq. (i) $\times$ 28 and (ii) Eq., we get

$$
\begin{aligned}
2A+6B &= 0 \\
-2A+B &= 1 \\
\hline
7B &= 1 \\
B &= \tfrac{1}{7}
\end{aligned}
$$

On putting in Eq. (i), we get

$A = -3 \times \frac{1}{7} = \frac{-3}{7}$

From Eqs. (iii) and (iv) $\times 3$, we get

$$
\begin{aligned}
C+3D &= 1 \\
-6C+3D &= 3 \\
+ \quad - \quad & - \\
7C &= -2
\end{aligned}
$$

$C = -\frac{2}{7} \quad D = \frac{3}{7}$

$$
A : C = -\frac{3}{7}; -\frac{2}{7} \Rightarrow 3:2
$$

and $B : D = 1 : 3$

Question75

If $x \in \left[2n\pi - \frac{\pi}{4}, 2n\pi + \frac{3\pi}{4}\right]$ and $n \in Z$, then $\int \sqrt{1-\sin 2x} dx =$

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Options:

A. $-\cos x + \sin x + c$

B. $\cos x + \sin x + c$

C. $-\cos x - \sin x + c$

D. $\cos x - \sin x + c$

Answer: B

Solution:

Simplify using trigonometric identities.

$$\int \sqrt{1-\sin 2x dx} \quad [\because \sin 2x = 2\sin x \cos x]$$

$$= \int \sqrt{\sin^2 x + \cos^2 x - 2\sin x \cos x dx}$$

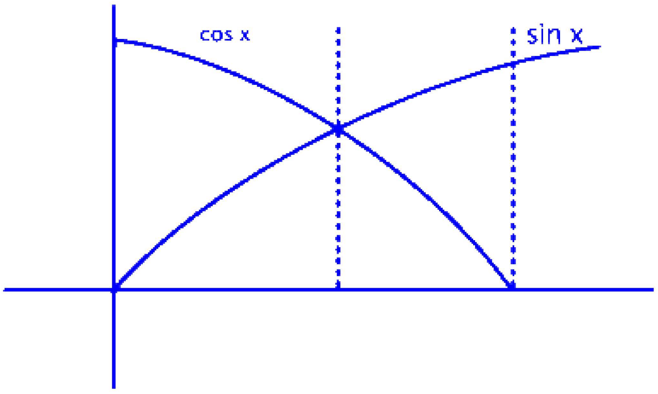
$$\therefore \therefore 1 = \sin^2 x + \cos^2 x]$$

$$= \int \sqrt{(\cos x - \sin x)^2} dx$$

$$\int (\cos x - \sin x) dx$$

$$= \sin x - (-\cos x) + c = \sin x + \cos x + c$$

$$= \cos x + \sin x + c$$



Therefore, the integral of $\sqrt{1-\sin 2x}$ w.r.t. x, if $\sin x + \cos x + c$. where c is the constant of integration.

Question76

$\int e^x \left(\frac{x+2}{x+4}\right)^2 dx =$

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Options:

A. $-\frac{xe^x}{(x+4)^2} + c$

B. $-\frac{xe^x}{(x+4)} + c$

C. $\frac{xe^x}{(x+4)} + c$

D. $\frac{2xe^x}{(x+4)} + c.$

Answer: C

Solution:

$$\int e^x \cdot \left(\frac{x+2}{x+4}\right)^2 dx$$

$$= \int e^x \frac{x^2+4x+4}{(x+4)^2} dx$$

$$= \int e^x \cdot \left[\frac{x}{x+4} + \frac{4}{(x+4)^2}\right] dx$$

We know that,

$$\int e^x(f(x)+f'(x))dx = e^x \cdot f(x)+c$$

$$= \int e^x\left[\frac{x}{x+4}+\frac{4}{(x+4)^2}\right]dx$$

$$= e^x \cdot \frac{x}{(x+4)}+c$$

Or

(c) Alternate solution

$$\int e^x\left(\frac{x+2}{x+4}\right)^2 dx$$

Let $u = x+4$

$du = dx; x = u-4$

$$\int e^{u-4}\left(\frac{u-4+2}{u}\right)^2 du$$

$$\int e^{u-4}\left(\frac{u-2}{u}\right)^2 dx \Rightarrow \int e^{u-4}\left(1-\frac{2}{u}\right)^2 du$$

$$\int e^{u-4}\left(1+\frac{4}{u^2}-\frac{4}{u}\right)dx$$

$$\int\left[e^{u-4}+4\frac{e^{u-4}}{u^2}-\frac{4e^{u-4}}{u}\right]du$$

$$e^{-4}\left[e^u-4\int\left(\frac{e^u}{u}-\frac{e^u}{u^2}\right)du\right]$$

But Eq. (i)

$e^{-4}\left[e^u-4\frac{e^u}{u}\right]+c$

On putting the value of $u = x+4$

$e^{-4}\left[e^{x+4}-\frac{4e^{x+4}}{x+4}\right]+c$

$$e^x-\frac{4e^x}{(x+4)}+c$$

$$= e^x\left(\frac{x+4-4}{x+4}\right)+c = \frac{xe^x}{x+4}+c$$

Question77

If $\int \frac{1}{1-\cos x}dx = \tan\left(\frac{x}{\alpha}+\beta\right)+c$, then one of the values of $\frac{\pi\alpha}{4}-\beta$ is

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Options:

A. $-\frac{\pi}{2}$

B. π

C. 0

D. $\frac{\pi}{4}$

Answer: B

Solution:

If $\int \frac{1}{1-\cos x} dx=\tan \left(\frac{x}{\alpha}+\beta\right)+c$

$\frac{\pi \alpha}{4}-\beta= ?$

$\int \frac{1}{1-\cos x} dx=\tan \left(\frac{x}{\alpha}+\beta\right)+c$

$\int \frac{1}{1-1+2 \sin ^2\left(\frac{x}{2}\right)} dx=\tan \left(\frac{x}{\alpha}+\beta\right)+c$

$\frac{1}{2} \int \operatorname{cosec}^2\left(\frac{x}{2}\right) dx=\tan \left(\frac{x}{\alpha}+\beta\right)+c\left(\because \sin ^2 \frac{x}{2}=\frac{1}{\operatorname{cosec}^2 \frac{x}{2}}\right)$

$\frac{1}{2} x-2 \cot \left(\frac{x}{2}\right)=\tan \left(\frac{x}{\alpha}+\beta\right)+c$

$-\cot \left(\frac{x}{2}\right)=\tan \left(\frac{x}{\alpha}+\beta\right)+c$

$-\cot \left(\frac{x}{2}\right)=\tan \left(\frac{x}{\alpha}+\beta\right)+c \quad\left(\because \cot y=\frac{1}{\tan y}\right)$

$\Rightarrow-\cot \left(\frac{x}{2}\right)=\tan \left(\frac{\pi}{2}-\frac{x}{2}\right)$

$\Rightarrow \tan \left(\frac{x}{2}-\frac{\pi}{2}\right)+C=\tan \left(\frac{x}{\alpha}+\beta\right)+c$

On comparison, we get

$-\frac{\pi}{2}+\frac{x}{2}=\frac{x}{\alpha}+\beta$

On solving these and we get

$\alpha=2, \beta=-\frac{\pi}{2}$

Now, the value of $\frac{\pi \alpha}{4}-\beta=\pi$

Question78

If $n \geq 2$ is a natural number and $0<\theta<\frac{\pi}{2}$, then $\int \frac{\left(\cos ^n \theta-\cos \theta\right)^{1 / n}}{\cos ^{n+1} \theta} \sin \theta d \theta=$

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Options:

A. $\frac{n}{n-1}\left(\cos ^{(1-n)} \theta-1\right)^2+c$

B. $\frac{n}{(n+1)(1-n)}\left(\cos ^{(1-n)} \theta-1\right)^{1+\frac{1}{n}}+c$

C. $\frac{1}{n-1}\left(\cos ^{(n-1)} \theta-1\right)^2+c$

D. $\frac{n}{1-n^2}\left(1-\cos ^{(1-n)} \theta\right)^{(n+1) / n}+c$

Answer: D

Solution:

$$I=\int \frac{\left(\cos^n \theta-\cos \theta\right)^{\frac{1}{n}}}{\cos^{n+1} \theta} \sin \theta d \theta$$

Let $=\int\left(\frac{\cos^n \theta-\cos \theta}{\cos^n \theta}\right)^{\frac{1}{n}} \frac{\sin \theta}{\cos^n \theta} d \theta$

$$=\int\left(1-\cos^{(1-m)} \theta\right)^{\frac{1}{n}} \cos^{(-n)} \theta \sin \theta d \theta$$

Let $t=1-\cos^{(1-n)} \theta$

$\Rightarrow \quad d t=-(1-n) \cos^{(-n)} \theta(-\sin \theta) d \theta$

$\Rightarrow \quad \frac{d t}{1-n}=\cos^{(-n)} \theta \sin \theta d \theta$

$\therefore \quad I=\frac{1}{1-n} \int(t)^{\frac{1}{n}} d t=\frac{1}{1-n} \frac{t^{\frac{1+n}{n}}}{\frac{1+n}{n}}+C$

$$=\frac{n}{1-n^2}\left(1-\cos^{(1-n)} \theta\right)^{\frac{(n+1)}{n}}+C$$

Question79

If $\frac{x^2+3}{x^4+2x^2+9}=\frac{Ax+B}{x^2+ax+b}+\frac{Cx+D}{x^2+cx+b}$, then $aA+bB+cC+D=$

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Options:

A. 1

B. 0

C. -1

D. 2

Answer: D

Solution:

Let

$$\frac{x^2+3}{x^4+2x^2+9}=\frac{x^2+3}{\left(x^2-2x+3\right)\left(x^2+2x+3\right)}$$

$$=\frac{Ax+B}{x^2-2x+3}+\frac{Cx+D}{x^2+2x+3}$$

$$=\frac{\left(x^2+2x+3\right)(Ax+B)+\left(x^2-2x+3\right)(Cx+D)}{\left(x^2-2x+3\right)\left(x^2+2x+3\right)}$$

On comparing the numerator, we get

$$\Rightarrow x^2+3=\left(x^2-2x+3\right)(Cx+D)+\left(x^2+2x+3\right)(Ax+B)$$

$$=x^3(A+C)+x^2(2A+B-2C+D)+x(3A+2B+3C-2D)+3B+3D$$

Comparing the like terms, we get

$$\begin{aligned} A+C&=0 \\ 2A+B-2C+D&=1 \\ 3A+2B+3C-2D&=0 \\ 3B+3D&=3 \end{aligned}$$

On solving these equations, we get

$$A=0, B=\frac{1}{2}, C=0 \text{ and } D=\frac{1}{2}$$

$$\therefore \quad \frac{x^2+3}{(x^2-2x+3)(x^2+2x+3)} = \frac{1}{2(x^2-2x+3)}+\frac{1}{2(x^2+2x+3)}$$

$$\therefore \quad a=-2, b=3 \text{ and } c=2$$

$$A=0, B=\frac{1}{2},$$

$$C=0, D=\frac{1}{2}$$

Thus, $aA+bB+cC+D$

$=(-2)(0)+\frac{1}{2}(3)+0+\frac{1}{2}=2$

Question80

$\int \frac{dx}{x(x^4+1)} =$

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Options:

A. $\log\left(\frac{x}{x^4+1}\right)+c$

B. $\frac{3}{4}\log\left(x^4+1\right)+c$

C. $\frac{1}{3}\log\left(\frac{x^3}{x^4+1}\right)+c$

D. $\frac{1}{4}\log\left(\frac{x^4}{x^4+1}\right)+c$

Answer: D

Solution:

$\int \frac{dx}{x(x^4+1)}$

Let $x^4=t$

then, $4x^3 dx = dt$

$$\int \frac{dx}{x\left(x^4+1\right)}=\int \frac{dt}{4x^3 \cdot x(t+1)}=\int \frac{dt}{4t(t+1)}$$
$$=\frac{1}{4}\int\left[\frac{-1}{(t+1)}+\frac{1}{t}\right]dt$$
$$=\frac{1}{4}[-\log|t+1|+\log|t|]+C$$
$$=\frac{1}{4}\log\left[\frac{t}{t+1}\right]+C$$
$$=\frac{1}{4}\log\left|\frac{x^4}{x^4+1}\right|+C, \quad (\because x^4=t)$$

Question81

$$\int \frac{dx}{\sqrt{\sin^3 x \cos(x-a)}}=$$

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Options:

A. $\frac{1}{\cos\alpha}\sqrt{\cot x+\tan\alpha}+c$

B. $\frac{1}{\sqrt{\cos\alpha}}\sqrt{\cot x-\tan\alpha}+c$

C. $\frac{-1}{\sqrt{\sin\alpha}}\sqrt{\cot x+\tan\alpha}+c$

D. $\frac{-2}{\sqrt{\cos\alpha}}\sqrt{\cot x+\tan\alpha}+c$

Answer: D

Solution:

$$Let I=\int \frac{dx}{\sqrt{\sin^3 x \cos(x-\alpha)}}$$
$$\therefore \quad I=\int \frac{dx}{\sqrt{\sin^4 x \frac{(\cos x \cos\alpha+\sin x \sin\alpha)}{\sin x}}}$$
$$=\int \frac{\operatorname{cosec}^2 x dx}{\sqrt{\cot x \cos^2+\sin\alpha}}$$

Let $\cot x=z$

$$\Rightarrow -\operatorname{cosec}^2 x dx=dz$$
$$=-\int \frac{dz}{\sqrt{\cos\alpha z+\sin\alpha}}$$
$$=-\int \frac{dz}{\sqrt{\cos\alpha(z+\tan\alpha)}}$$

$=-\frac{1}{\sqrt{\cos\alpha}}\int \frac{dz}{\sqrt{z+\tan\alpha}}$

Let $z+\tan\alpha=t^2$

$$\Rightarrow dz = 2tdt = \frac{-2}{\sqrt{\cos\alpha}} \int \frac{tdt}{\sqrt{t^2}}$$
$$= \frac{-2}{\sqrt{\cos\alpha}} \int dt = \frac{-2}{\sqrt{\cos\alpha}} \cdot t + c$$
$$= \frac{-2}{\sqrt{\cos\alpha}} \cdot \sqrt{z+\tan\alpha} + c$$
$$= \frac{-2\sqrt{\cot x + \tan\alpha}}{\sqrt{\cos\alpha}} + c$$

Question82

$\int \frac{e^{2x}}{\sqrt[4]{e^x+1}} dx =$

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Options:

A. $\frac{4}{7}(e^x+1)^{\frac{4}{3}}(3e^x-1)+c$

B. $\frac{2}{21}(e^x+1)^{\frac{3}{4}}(3e^x-7)+c$

C. $\frac{4}{21}(e^x+1)^{3/4}(3e^x-4)+c$

D. $\frac{8}{21}(e^x+1)^{3/4}(3e^x-1)+c$

Answer: C

Solution:

Let $I = \int \frac{e^{2x}}{\sqrt[4]{e^x+1}} dx$

Let $(e^x+1)^{\frac{1}{4}} = t$

$$\Rightarrow \quad e^x = t^4 - 1$$
$$\Rightarrow \quad e^{2x} = (t^4-1)^2$$
$$\Rightarrow 2e^{2x} \cdot dx = 2(t^4-1) \cdot 4t^3 \cdot dt$$
$$I = \int \frac{(t^4-1) \cdot 4t^3 dt}{t}$$
$$= 4\int t^2(t^4-1) dt$$
$$= 4\left[\frac{t^7}{7} - \frac{t^3}{3}\right] + C$$
$$= 4t^3\left[\frac{t^4}{7} - \frac{1}{3}\right] + C$$
$$= \frac{4}{21} t^3 [3t^4 - 1] + C$$
$$= \frac{4}{21}(e^x+1)^{\frac{3}{4}}(3e^x-4) + C$$

Question83

$\int \frac{2-\sin x}{2 \cos x+3} d x=$

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Options:

A. $\frac{2}{\sqrt{5}} \tan ^{-1}\left(\frac{1}{\sqrt{3}} \tan \frac{x}{2}\right)-\log \sqrt{2 \cos x+3}+c$

B. $\frac{4}{\sqrt{5}} \tan ^{-1}\left(\frac{1}{\sqrt{5}} \tan \frac{x}{2}\right)+\log \sqrt{2 \cos x+3}+c$

C. $\frac{3}{\sqrt{5}} \tan ^{-1}\left(\frac{1}{\sqrt{5}} \tan \frac{x}{2}\right)+\log \sqrt{2 \cos x-3}+c$

D. $\frac{1}{\sqrt{5}} \tan ^{-1}\left(\frac{1}{\sqrt{5}} \tan \frac{x}{3}\right)-\log \sqrt{2 \cos x-3}+c$

Answer: B

Solution:

$\int \frac{2-\sin x}{2 \cos x+3} d x$

$=\int \frac{2}{2 \cos x+3} d x-\int \frac{\sin x}{2 \cos x+3}$

$=2 \int \frac{\sec ^2\left(\frac{x}{2}\right)}{\tan ^2 \frac{x}{2}+5} d x+\frac{1}{2} \int \frac{1}{u} d u$

$\left[\begin{array}{l}\text{let } 2 \cos x+3=u \\ \Rightarrow-2 \sin x d x=d\end{array}\right]$

$=2 \int \frac{2 \sqrt{5}}{5 t^2+5} d t+\frac{1}{2} \log |u|$

$\left[\text{Let } \frac{1}{\sqrt{5}} \tan \left(\frac{x}{2}\right)=t \Rightarrow \frac{1}{2 \sqrt{5}} \sec ^2 \frac{x}{2} d x=d x\right]$

$=\frac{4 \sqrt{5}}{5} \int \frac{d t}{t^2+1}+\frac{1}{2} \log |2 \cos x+3|$

$=\frac{4}{\sqrt{5}} \tan ^{-1} \frac{t}{1}+\frac{\log |2 \cos x+3|}{2}+C$

$=\frac{4}{\sqrt{5}} \tan ^{-1}\left(\frac{\tan \frac{x}{2}}{\sqrt{5}}\right)+\frac{\log (2 \cos x+3)}{2}+C$

Question84

$\int \sin ^{-1} \sqrt{\frac{x}{a+x}} d x=$

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Options:

A. $(a+x) \tan ^{-1} \sqrt{\frac{x}{a}}-\sqrt{a x}+c$

B. $\frac{1}{a+x}\tan^{-1}\left(\frac{x}{a}\right)-\sqrt{ax}+c$

C. $(a+x)\tan^{-1}\left(\frac{a}{x}\right)+\sqrt{ax}+c$

D. $\sqrt{a+x}\tan^{-1}\left(\frac{x}{a}\right)+ax+c$

Answer: A

Solution:

$I=\int\sin^{-1}\sqrt{\frac{x}{a+x}}dx$

Let $x=a\tan^2 t$

$\Rightarrow dx=2a\tan t\sec^2 tdt$

$I=\int\sin^{-1}\sqrt{\frac{a\tan^2 t}{a+a\tan^2 t}}\left(2a\tan t\sec^2 t\right)dt$

$=2a\int\sin^{-1}\sqrt{\frac{a\tan^2 t}{a\sec^2 t}}\tan t\sec^2 tdt$

$=2a\int t\left(\tan t\sec^2 t\right)dt$

$=2a\left[t\int\tan t\sec^2 tdt-\int\tan t\sec^2 tdt\right]$

$=2a\left[\frac{t\tan^2 t}{2}-\int\frac{\tan^2 t}{2}dt\right]$

$=at\tan^2 t-a\tan t+at+C$

$=a\left[\frac{x}{a}\tan^{-1}\left(\sqrt{\frac{x}{a}}\right)-\sqrt{\frac{x}{a}}+\tan^{-1}\sqrt{\frac{x}{a}}\right]+C=(a+x)\tan^{-1}\sqrt{\frac{x}{a}}-\sqrt{ax}+C$

Question85

If $\frac{A}{x-a}+\frac{Bx+C}{x^2+b^2}=\frac{1}{(x-a)(x^2+b^2)}$, then C =

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Options:

A. $\frac{-1}{a^2+b^2}$

B. $\frac{1}{a^2+b^2}$

C. $\frac{-a}{a^2+b^2}$

D. $\frac{a}{a^2+b^2}$

Answer: C

Solution:

We have, $\frac{A}{x-a}+\frac{Bx+C}{x^2+b^2}=\frac{1}{(x-a)(x^2+b^2)}$

$$\Rightarrow \quad \frac{A\left(x^2+b^2\right)+(Bx+C)(x-a)}{(x-a)\left(x^2+b^2\right)}=\frac{1}{(x-a)\left(x^2+b^2\right)}$$
$$\Rightarrow A\left(x^2+b^2\right)+(Bx+C)(x-a)=1$$
$$\Rightarrow Ax^2+Ab^2+Bx^2-Bxa+Cx-Ca=1$$
$$\Rightarrow (A+B)x^2+(C-Ba)x+\left(Ab^2-Ca\right)=1$$

On comparing constants and variables, we get

$$A+B=0 \Rightarrow B=-A$$
$$C-Ba=0 \Rightarrow C-(-A)a=0 \Rightarrow C=-Aa$$
$$Ab^2-Ca=1$$
$$\Rightarrow \quad Ab^2-(-Aa)a=1 \Rightarrow Ab^2+Aa^2=1$$
$$\Rightarrow \quad A\left(b^2+a^2\right)=1 \Rightarrow A=\frac{1}{a^2+b^2}$$

Therefore, $C=-Aa=\frac{-a}{a^2+b^2}$

Question86

$\int \frac{2x^2-3}{(x^2-4)(x^2+1)}dx = A\tan^{-1}x + B\log(x-2) + C\log(x+2)$, **then** $6A+7B-5C=$

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Options:

A. 9

B. 10

C. 6

D. 8

Answer: A

Solution:

Let $I=\int \frac{2x^2-3}{(x^2-4)(x^2+1)}dx$

$$=\int \frac{\left(x^2-4\right)}{\left(x^2-4\right)\left(x^2+1\right)}+\frac{\left(x^2+1\right)}{\left(x^2-4\right)\left(x^2+1\right)}dx$$
$$=\int\left(\frac{1}{x^2+1}+\frac{1}{x^2-4}\right)dx$$
$$=\tan^{-1}x+\int \frac{1}{(x-2)(x+2)}dx$$
$$=\tan^{-1}x+\frac{1}{4}\int\left(\frac{1}{x-2}-\frac{1}{x+2}\right)dx$$
$$=\tan^{-1}x+\frac{1}{4}\log(x-2)-\frac{1}{4}\log(x+2)+C$$

Here, $A=1, B=\frac{1}{4}$ and $C=-\frac{1}{4}$

$$6A \rightarrow 7B - 5C = 6 + \frac{7}{4} - 5\left(-\frac{1}{4}\right)$$
$$= 6 + \frac{7}{4} + \frac{5}{4} = 9$$

Question87

$\int \frac{3x^9+7x^8}{(x^2+2x+5x^8)^2} dx =$

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Options:

A. $\frac{x^7}{5x^7+x+2} + c$

B. $\frac{x^7}{2(5x^7+x+2)} + c$

C.

$\frac{1}{2(5x^7+x+2)} + c$

D. $\frac{-x^7}{2(5x^7+x+2)} + c$

Answer: B

Solution:

We have $I = \int \frac{3x^9 + 7x^8}{(x^2 + 2x + 5x^8)^2} dx$

$= \int \frac{3x^{-7} + 7x^{-8}}{(x^{-6} + 2x^{-7} + 5)^2} dx \; [\because \text{ divide by } x^{16}]$

Let $x^{-6} + 2x^{-7} + 5 = t$

$(-6x^{-7} - 14x^{-8}) dx = dt$

$(3x^{-7}+7x^{-8}) dx = -\frac{1}{2} dt$

$$I = -\frac{1}{2} \int \frac{1}{t^2} dt = \frac{1}{2t} + C$$
$$= \frac{1}{2(x^{-6} + 2x^{-7} + 5)} + C$$
$$= \frac{x^7}{2(x + 2 + 5x^7)} + C$$

Question88

$\int \frac{\cos x + x \sin x}{x(x+\cos x)} dx =$

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Options:

A. $\log\left|x^2+x\cos x\right|+c$

B. $\log\left|\frac{x}{x+\cos x}\right|+c$

C. $\log\left|\frac{\cos x}{x+\cos x}\right|+c$

D. $\log\left|\frac{1}{x+\cos x}\right|-\log x+c$

Answer: B

Solution:

$$
\begin{aligned}
&\text{We have } I=\int\frac{\cos x+x\sin x}{x(x+\cos x)}dx\\
&=\int\frac{(x+\cos x)-x(1-\sin x)}{x(x+\cos x)}dx\\
&=\int\left(\frac{1}{x}-\frac{1-\sin x}{x+\cos x}\right)dx\\
&=\log|x|-\log(x+\cos x)+C\\
&=\log\left|\frac{x}{x+\cos x}\right|+C
\end{aligned}
$$

Question89

If $\int\sqrt{\frac{2}{1+\sin x}}dx=2\log|A(x)-B(x)|+C$ **and** $0\leq x\leq\frac{\pi}{2}$**, then** $B\left(\frac{\pi}{4}\right)=$

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Options:

A. $\frac{1}{\sqrt{2+3\sqrt{3}}}$

B. $\frac{1}{\sqrt{3+2\sqrt{2}}}$

C. $\frac{-1}{\sqrt{3+2\sqrt{2}}}$

D. $\frac{2}{\sqrt{2+\sqrt{2}}}$

Answer: B

Solution:

We have $I=\int \sqrt{\frac{2}{1+\sin x}} d x$

$=\int \sqrt{\frac{2}{\frac{1+2 \tan x / 2}{1+\tan ^2 x / 2}}} d x$

$=\int \sqrt{\frac{2 \sec ^2 x / 2}{1+\tan ^2 x / 2+2 \tan x / 2}} d x$

$=\sqrt{2} \int \frac{\sec x / 2}{1+\tan x / 2} d x$

$=\sqrt{2} \int \frac{1}{\cos \frac{x}{2}+\sin \frac{x}{2}} d x$

$=\int \frac{1}{\frac{1}{\sqrt{2}} \cos \frac{x}{2}+\frac{1}{\sqrt{2}} \sin \frac{x}{2}} d x$

$=\int \frac{1}{\sin \left(\frac{\pi}{4}+\frac{x}{2}\right)} d x=\int \operatorname{cosec}\left(\frac{\pi}{4}+\frac{x}{2}\right) d x$

$=2 \log \left\lvert\, \operatorname{cosec}\left(\frac{\pi}{4}+\frac{x}{2}\right)-\cot \left(\frac{\pi}{4}+\frac{x}{2}\right)+C\right.$

$B(x)=\cot \left(\frac{\pi}{4}+\frac{x}{2}\right)$

$B\left(\frac{\pi}{4}\right)=\cot \left(\frac{\pi}{4}+\frac{\pi}{8}\right)=\cot \frac{3 \pi}{8}=\frac{1}{\sqrt{3+2 \sqrt{2}}}$

Question90

If $\int \frac{3}{2 \cos ^3 x \sqrt{2 \sin 2 x}} d x=\frac{3}{2}(\tan x)^B+\frac{3}{10}(\tan x)^A+C$, than $A=$

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Options:

A. $\frac{1}{2}$

B. 1

C. 5

D. $\frac{5}{2}$

Answer: D

Solution:

We begin with the integral:

$I=\int \frac{3}{2 \cos ^3 x \sqrt{2 \sin 2 x}} d x$

Let's simplify this expression:

$I = \frac{3}{2} \int \frac{\sec^3 x}{\sqrt{4 \sin x \cos x}} \, dx$

This can be further rewritten as:

$I = \frac{3}{4} \int \frac{\sec^2 x \times \sec^2 x \, dx}{\sqrt{\tan x}}$

Now, make the substitution $\tan x = t^2$. Then, the differential $\sec^2 x \, dx$ becomes:

$\sec^2 x \, dx = 2t \, dt$

Substituting back, the integral becomes:

$I = \frac{3}{4} \times 2 \int \frac{1}{t} \times (1 + t^4) t \, dt$

Simplifying further, we have:

$I = \frac{3}{2} \int (1 + t^4) \, dt$

Evaluating this integral:

$I = \frac{3}{2} t + \frac{3}{2} \times \frac{t^5}{5} + C$

Substituting back for $t = \sqrt{\tan x}$:

$I = \frac{3}{2} (\tan x)^{1/2} + \frac{3}{10} (\tan x)^{5/2} + C$

From this result, we identify that $B = \frac{1}{2}$ and $A = \frac{5}{2}$.

Question91

If $\frac{1}{x^4+1} = \frac{Ax+B}{x^2+\sqrt{2}x+1} + \frac{Cx+D}{x^2-\sqrt{2}x+1}$, then $BD - AC =$

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Options:

A. $\frac{3}{8}$

B. $\frac{1}{8}$

C. 1

D. 0

Answer: A

Solution:

To solve for $BD - AC$ given the equation $\frac{1}{x^4+1} = \frac{Ax+B}{x^2+\sqrt{2}x+1} + \frac{Cx+D}{x^2-\sqrt{2}x+1}$, we follow these steps:

Starting with:

$\frac{1}{x^4+1} = \frac{(Ax+B)(x^2-\sqrt{2}x+1)+(Cx+D)(x^2+\sqrt{2}x+1)}{(x^2+\sqrt{2}x+1)(x^2-\sqrt{2}x+1)}$

Simplify the denominator:

$(x^2 + \sqrt{2}x + 1)(x^2 - \sqrt{2}x + 1) = (x^2 + 1)^2 - (\sqrt{2}x)^2 = x^4 + 1$

Thus, we expand the numerator:

$Ax^3 - \sqrt{2}Ax^2 + Ax + Bx^2 - \sqrt{2}Bx + B + Cx^3 + \sqrt{2}Cx^2 + Cx + Dx^2 + \sqrt{2}Dx + D$

Collecting like terms:

x^3 terms: $A + C = 0$

x^2 terms: $-\sqrt{2}A + B + \sqrt{2}C + D = 0$

x terms: $A - \sqrt{2}B + C + \sqrt{2}D = 0$

Constant terms: $B + D = 1$

From the equations:

$A + C = 0$

$-\sqrt{2}A + B + \sqrt{2}C + D = 0$

$A - \sqrt{2}B + C + \sqrt{2}D = 0$

$B + D = 1$

From equation 4, if $B = D = \frac{1}{2}$, substituting into equations, we derive:

From equation 1, $C = -A$.

Substitute into equation 2:

$-\sqrt{2}A + B + \sqrt{2}(-A) + D = 0$

This becomes:

$-2\sqrt{2}A + \frac{1}{2} + \frac{1}{2} = 0 \implies -2\sqrt{2}A = -1 \implies A = \frac{1}{2\sqrt{2}}$

Then $C = -\frac{1}{2\sqrt{2}}$.

Now calculate $BD - AC$:

$BD - AC = \left(\frac{1}{2} \times \frac{1}{2}\right) - \left(\frac{1}{2\sqrt{2}} \times \left(-\frac{1}{2\sqrt{2}}\right)\right) = \frac{1}{4} + \frac{1}{8} = \frac{3}{8}$

Question92

$\int \frac{2x^2 \cos x^2 - \sin x^2}{x^2} dx =$

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Options:

A. $\frac{\sin x^2}{x^2} + c$

B. $\frac{\cos x^2}{x^2} + c$

C. $\sin x^2 + c$

D. $\frac{\sin x^2}{x} + c$

Answer: D

Solution:

We know,

$$\int [xf'(x) + f(x)]dx = xf(x) + C$$

We have, $\int \frac{2x^2 \cos x^2 - \sin x^2}{x^2} dx$

$$= \int \frac{x \times \cos x^2 \times 2x - \sin x^2}{x^2} dx$$

$=$

$$= \int \frac{x(\sin x^2)' - (\sin x^2)}{x^2} dx$$

$=$

$$= \frac{\sin x^2}{x} + C$$

Question93

If $\int \frac{\log(1+x^4)}{x^3} dx = f(x) \log\left(\frac{1}{g(x)}\right) + \tan^{-1}(h(x)) + c$**, then** $h(x)\left[f(x) + f\left(\frac{1}{x}\right)\right] =$

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Options:

A. $h(x)g(-x)$

B. $\frac{g(x)}{2}$

C. $g(x) + g(-x)$

D. $g(x)h(x)$

Answer: B

Solution:

To evaluate the given integral, we start with:

$I = \int \frac{\log(1+x^4)}{x^3} dx$

Applying integration by parts, let's simplify:

$= \log(1 + x^4) \int x^{-3} dx - \int \left(\frac{d}{dx}(\log(1 + x^4)) \cdot \int x^{-3} dx\right) dx$

The derivative $\frac{d}{dx}\log(1 + x^4) = \frac{4x^3}{1+x^4}$, so the expression becomes:

$= \frac{\log(1+x^4)}{-2x^2} + \int \frac{2x}{1+x^4} dx$

The next part simplifies further:

$= -\frac{1}{2x^2}\log(1 + x^4) + \tan^{-1}(x^2) + C$

Rewriting the logarithm expression:

$= \frac{1}{2x^2} \log\left(\frac{1}{1+x^4}\right) + \tan^{-1}(x^2) + C$

We identify the parts:

$f(x) = \frac{1}{2x^2}$

$g(x) = 1 + x^4$

$h(x) = x^2$

Now, consider:

$h(x)\left[f(x) + f\left(\frac{1}{x}\right)\right] = x^2\left[\frac{1}{2x^2} + \frac{x^2}{2}\right]$

Simplifying this expression:

$= x^2\left[\frac{1+x^4}{2x^2}\right] = \frac{g(x)}{2}$

Question94

Let $f(x) = \int \frac{x}{(x^2+1)(x^2+3)} dx$. If $f(3) = \frac{1}{4} \log\left(\frac{5}{6}\right)$, then $f(0) =$

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Options:

A. $\frac{1}{4} \log\left(\frac{1}{3}\right)$

B. 0

C. $\frac{1}{2} \log\left(\frac{1}{3}\right)$

D. $\log\left(\frac{1}{3}\right)$

Answer: A

Solution:

$(f(x) = \int \frac{x}{(x^2+1)(x^2+3)} dx = I$ (let) Let $x^2 = t, 2x dx = dt$

$$
\begin{aligned}
I &= \frac{1}{2}\int \frac{1}{(t+1)(t+3)}dt \\
&= \frac{1}{4}\int\left(\frac{1}{t+1}-\frac{1}{t+3}\right)dt \\
&= \frac{1}{4}\log\left|\frac{t+1}{t+3}\right| + \log C \\
&= \frac{1}{4}\log\left|\frac{x^2+1}{x^2+3}\right| + \log c \\
&= \frac{1}{4}\left(\log\left(\frac{(x^2+1)c^4}{x^2+3}\right)\right) \\
f(3) &= \frac{1}{4}\log\left|\frac{10}{12}c^4\right| = \frac{1}{4}\log\frac{5}{6} \\
c^4 &= \frac{5}{6}\times\frac{12}{10} = 1 \Rightarrow c = 1 \\
f(0) &= \frac{1}{4}\log\left(\frac{1}{3}\right)
\end{aligned}
$$

Question95

$$\int \frac{2\cos 2x}{(1+\sin 2x)(1+\cos 2x)}dx =$$

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Options:

A. $2\tan x + \log(1+\tan x) + c$

B. $\tan x - 2\log(1+\tan x) + c$

C. $2\log(1+\tan x) + \tan x + c$

D. $2\log(1+\tan x) - \tan x + c$

Answer: D

Solution:

$$\text{Let } I = \int \frac{2\cos 2x}{(1+\sin 2x)(1+\cos 2x)} dx$$
$$= \int \frac{2 \times \left(2\cos^2 x - 1\right) dx}{(\sin x + \cos x)^2 \times 2\cos^2 x}$$
$$= \int \frac{\left(2\cos^2 x - 1\right)}{(\sin x + \cos x)^2} \sec^2 x dx$$
$$= \int \frac{\left(2 - \sec^2 x\right)}{(1+\tan x)^2} \sec^2 x dx$$
$$= \int \frac{\left(1 - \tan^2 x\right)}{(1+\tan x)^2} \sec^2 x dx$$
$$= \int \left(\frac{1-\tan x}{1+\tan x}\right) \sec^2 x dx$$
$$= \int \left(\frac{1-t}{1+t}\right) dt [\because \tan x = 1]$$
$$= -\int \left(\frac{t-1}{1+t}\right) dt = -\int \left(1 - \frac{2}{1+t}\right) dt$$
$$= -t + 2\log(1+t) + C$$
$$= -\tan x + 2\log(1+\tan x) + C$$

Question96

$$\int \left(\frac{x}{x\cos x - \sin x}\right)^2 dx =$$

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Options:

A. $\frac{x \operatorname{cosec} x}{x\cos x - \sin x} + \cot x + c$

B. $\frac{x \operatorname{cosec} x}{x\cos x - \sin x} - \cot x + c$

C. $\frac{x \operatorname{cosec} x}{x\cos + \sin x} + \cot x + c$

D. $\frac{x}{x\cos x - \sin x} - \cot x + c$

Answer: B

Solution:

$$\text{Let } I = \int \left(\frac{x}{x\cos x - \sin x}\right)^2 dx$$
$$= \int \frac{x^2}{(x\cos x - \sin x)^2} dx$$
$$= \int (x \operatorname{cosec} x) \cdot \frac{(x\sin x)}{(x\cos x - \sin x)^2} \ldots (i)$$

Let, $I_1 = \int \frac{x\sin x}{(x\cos x - \sin x)^2} dx$

Let, $x\cos x - \sin x = t \Rightarrow x\sin x dx = -d$

$$\Rightarrow \quad = -\int \frac{1}{t^2} dt$$
$$= \frac{1}{x\cos x - \sin x}$$

From Eq. (i), Integrate I, we get

$$I = (x \operatorname{cosec} x) \int \frac{x \sin x dx}{(x\cos x - \sin x)^2}$$
$$- \int \left[(x \operatorname{cosec} x)^2 \int \frac{x \sin x dx}{(x\cos x - \sin x)^2} \right] dx$$
$$= \frac{x \operatorname{cosec} x}{x\cos x - \sin x}$$
$$- \int \left[\frac{-x \operatorname{cosec} x \cdot \cot x + \operatorname{cosec} x}{x\cos x - \sin x} \right] dx$$
$$= \frac{x \operatorname{cosec} x}{x\cos x - \sin x} + \int \frac{1}{\sin^2 x} dx + C$$
$$= \frac{x \operatorname{cosec} x}{x\cos x - \sin x} - \cot x + C$$

Question97

$$\int \frac{1}{x^5 \sqrt[5]{x^5 + 1}} dx =$$

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Options:

A. $\frac{4}{\sqrt{x^5+1}} + c$

B. $4x^4 \left(x^5 + 1\right)^{4/3} + 0$

C.

$$-\frac{(x^5 + 1)^{4/5}}{4x^4} + c$$

D. $-\frac{\left(x^5+1\right)^{45}}{4x^5} + c$

Answer: C

Solution:

We have,

Iet $I = \int \frac{1}{x^3 (x^3 + 1)^{1/5}} dx$

$$= \int \frac{1}{x^2 \left(1 + \frac{1}{x^3}\right)^{1/5}} dx$$

Let $1 + \frac{1}{x^3} = t^3$

$-\frac{5}{x^6}dx = 5t^4dt \Rightarrow \frac{1}{x^6}dx = -t^4dt$

$$\begin{aligned} I &= -\int \frac{1}{t}t^4dt \\ &= -\int t^3dt = -\frac{t^4}{4} + C \\ &= -\frac{1}{4}\left[1+\frac{1}{x^5}\right]^{4/5} + C \\ &= -\frac{1}{4}\frac{\left(x^5+1\right)^{4/5}}{x^4} + C \end{aligned}$$

Question98

$\int \frac{x+1}{\sqrt{x^2+x+1}}dx =$

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Options:

A. $\frac{1}{2}\sqrt{x^2+x+1} + \frac{1}{2}\cosh^{-1}\left(\frac{x+2}{\sqrt{3}}\right) + c$

B. $\frac{1}{2}\sqrt{x^2+x+1} + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + c$

C. $\sqrt{x^2+x+1} + \frac{2}{\sqrt{3}}\log\left|x^2+x+1\right| + c$

D. $\sqrt{x^2+x+1} + \frac{1}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + c$

Answer: D

Solution:

Let $t = \int \frac{x+1}{\sqrt{x^2+x+1}}dx$

$= \frac{1}{2}\int \frac{2x+1}{\sqrt{x^2+x+1}}dx + \frac{1}{2}\int \frac{1}{\sqrt{x^2+x+1}}dx$

$I = I_1 + I_2(Let) \quad \ldots$ (i)

Let $I_1 = \frac{1}{2}\int \frac{2x+1}{\sqrt{x^2+x+1}}dx$

Let $x^2+x+1 = t^2$

$(2x+1)dr = 2dt$

$$\begin{aligned} t_1 &= \frac{1}{2}\int^{\frac{1}{t}} \frac{xdt}{} \\ &= \int dt = t = \sqrt{x^2+x+1} \\ I_2 &= \frac{1}{2}\int \frac{1}{\sqrt{x^2+x+1}}dr \\ &= \frac{1}{2}\int \frac{1}{\sqrt{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}}dx \end{aligned}$$

Let $x+\frac{1}{2}=\cos dx=dx$

$$f_2=\frac{1}{2}\int\frac{1}{\sqrt{N^2+\frac{3}{4}}}$$
$$=\frac{1}{2}\sinh^{-1}\left(\frac{\pi}{\sqrt{3}}\right)$$
$$=\frac{1}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Hence.

$$I=\sqrt{x^2+x+1}+\frac{1}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+C$$

Question99

$\int\left(\tan^9 x+\tan x\right)dx=0$

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Options:

A. $\frac{\tan^2 x}{12}\left(2\tan^3 x-3\tan^2 x+6\right)+c$

B. $\frac{\tan^2 x}{6}-\frac{\tan^5 x}{4}+\frac{\tan^2 x}{2}+c$

C. $\frac{\tan^2 x^2}{6}\tan^4 x+3\tan^2 x+4\Big)+c$

D. $\frac{\tan x}{12}\tan^4 x-3\tan^2 x+6+c$

Answer: A

Solution:

$$\text{Let } l=\int\left(\tan^3 x+\tan x\right)dx$$
$$=\int\tan x\left(\tan^6 x+1\right)dx$$
$$=\int\tan x\left(\left(\tan^2 x\right)^3+1^3\right)dx$$
$$=\int\tan^x\left(\tan^2 x+1\right)\left(\tan^4 x+1-\tan^2 x\right)dx$$
$$=\int\left(\tan^3 x-\tan^3 x+\tan x\right)\sec^3 x dx$$

Let $\tan x=t$

$\sec^2 x, dx=dt$

$$I=\int\left(t^5-t^3+t\right)dt$$
$$=\frac{t^6}{6}-\frac{t^4}{4}+\frac{t^2}{2}+C=\frac{t^2\left(2t^3-3t^2+6\right)}{12}+C$$
$$=\frac{\tan^2 x}{12}\left(2\tan^3 x-3\tan^2 x+6\right)+C$$

Question100

$\int \frac{\operatorname{cosec} x}{3 \cos x+4 \sin x} d x=$

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Options:

A. $\frac{1}{2} \log \left|\frac{\cos x}{3 \sin x+4 \cos x}\right|+c$

B. $\frac{1}{3} \log \left|\frac{\sin x}{3 \cos x+4 \sin x}\right|+c$

C. $\frac{1}{3} \log \left|\frac{3 \cos x+\sin x}{3 \cos x+4 \sin x}\right|+c$

D. $\frac{1}{2} \log \left|\frac{\cos x+4 \sin x}{3 \cos x+4 \sin x}\right|+c$

Answer: B

Solution:

Let

$$\begin{aligned} I & =\int \frac{\operatorname{cosec} x}{3 \cos x+4 \sin x} d x \\ & =\int \frac{\operatorname{cosec}^2 x}{3 \cot x+4} d x \end{aligned}$$

Let $3 \cot x+4=t$

$-3 \operatorname{cosec}^2 x d x=d t$

$\operatorname{cosec}^2 x d x=\frac{-1}{3} d t$

$$I=-\frac{1}{3} \int \frac{1}{t} d t$$

$$=\frac{-1}{3} \log |t|+C=\frac{1}{3} \log \left|\frac{1}{t}\right|+C$$

$$=\frac{1}{3} \log \left|\frac{1}{3 \cot x+4}\right|+C$$

$$=\frac{1}{3} \log \left|\frac{\sin x}{3 \cos x+4 \sin x}\right|+C$$

Question101

$\int e^{2 x+3} \sin 6 x d x=$

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Options:

A. $\frac{e^{2 x+3}}{40}(2 \sin (x x+6 \cos 6 x)+c$

B. $\frac{e^{2v+3}}{40}(2\cos 6x + 6\sin 6x) + c$

C. $\frac{e^{2n+3}}{20}(\sin 6x - 3\cos 6x) + c$

D. $\frac{e^{2n3}}{20}(\cos 8x - 3\sin 6x) + c$

Answer: C

Solution:

Let $I = \int e^{2x+3} \sin 6x dx$

$$= \frac{e^{2x+3}\sin 6x}{2} - \int 3e^{2x+3} \cdot \cos 6x dx$$
$$= \frac{e^{2x+3}\sin 6x}{2}$$
$$- \frac{\left[\frac{3e^{2x+3}\cos 6x}{2} - \int -9e^{2x+3}\sin 6x dx\right]}{2} = \frac{e^{2x+3}\sin 6x}{2} - \frac{3e^{2x+3}\cos 6x}{2} - 9I$$
$$\Rightarrow 10I = \frac{e^{2x+3}(\sin 6x - 3\cos 6x)}{2}$$
$$\therefore \quad I = \frac{e^{2x+3}(\sin 6x - 3\cos 6x)}{20} + C$$

Question102

$\frac{2x^2+1}{x^3-1} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1} \Rightarrow 7A + 2B + C =$

AP EAPCET 2022 - 5th July Morning Shift

Options:

A. 8

B. 9

C. 10

D. 11

Answer: B

Solution:

Here, $\frac{2x^2+1}{x^3-1} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$

Resolving into partial fractions,

$\frac{2x^2+1}{x^3-1} = \frac{A(x^2+x+1)+(Bx+C)(x-1)}{(x-1)(x^2+x+1)}$

$$\Rightarrow \quad 2x^2 + 1 = Ax^2 + Ax + A + Bx^2 - Bx + Cx - C$$
$$\Rightarrow \quad 2x^2 + 1 = x^2(A + B) + x(A - B + C) + A - C$$

Comparing with coefficient, we get

$A + B = 2, A - B + C = 0$ and $A - C = 1$

Clearly, $A = 1, B = 1$ and $C = 0$

So, $7A + 2B + C = 7(l) + 2(l) + 0 = 9$

Question103

$\int \frac{3x+4}{x^3-2x+4} dx = \log f(x) + C \Rightarrow f(3) =$

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Options:

A. $\frac{1}{\sqrt{17}}$

B. $\frac{1}{17}$

C. $\frac{2}{15}$

D. $\frac{2}{17}$

Answer: A

Solution:

$$\int \frac{3x + 4}{x^3 - 2x + 4} dx = \log f(x) + C$$
$$I = \int \frac{3x + 4}{x^3 - 2x + 4}$$

$$\text{Since, } x^3 - 2x - 4 = (x - 2)\left(x^2 + 2x + 2\right)$$
$$\frac{3x + 4}{x^3 - 2x + 4} = \frac{A}{(x - 2)} + \frac{Bx + C}{(x^2 + 2x + 2)}$$
$$\Rightarrow 3x + 4 = A\left(x^2 + 2x + 2\right) + (Bx + C)(x - 2)$$

On putting $x = 2, A = 1$ and comparing coefficients of x^2, we get

$0 = A + B \Rightarrow B = -1$

On putting $x = 0$, we have $2A - 2C$

$\Rightarrow \quad C = -1$

$$\int \frac{3x+4}{x^3-2x+4} dx = \int \frac{dx}{x-2} - \int \frac{x+1}{x^2+2x+2} dx$$
$$= \log |x - 2| - \frac{1}{2} \log \left|\left(x^2 + 2x + 2\right)\right| + C$$
$$= \log f(x) + C = \log \frac{|x-2|}{\sqrt{(x^2+2x+2)}} + C$$

$$f(x)=\frac{|x-2|}{\sqrt{(x^2+2x+2)}}$$

$$\text{So, } f(3)=\frac{|3-2|}{\sqrt{((3)^2+2(3)+2)}}$$

$$f(3)=\frac{1}{\sqrt{9+6+2}}=\frac{1}{\sqrt{17}}$$

Question104

$$\int \frac{e^{\tan^{-1} x}}{1+x^2}\left[\left(\sec^{-1} \sqrt{1+x^2}\right)^2+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right] dx=$$

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Options:

A. $e^{\tan^{-1} x}\left(\tan^{-1} x\right)^2+C$

B. $e^{\tan^{-1} x}\left(\sec^{-1} x\right)^2+C$

C. $e^{\tan^{-1} x}\left(\sec^{-1}\left(\sqrt{1+x^2}\right)\right)+C$

D. $e^{\tan^{-1}} \times\left(\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right)+C$

Answer: A

Solution:

Let

$I=\int \frac{e^{\tan^{-1}(x)}}{1+x^2}\left[\left(\sec^{-1} \sqrt{1+x^2}\right)^2+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right] dx$

On putting $\tan^{-1} x=t$ and $\frac{dx}{1+x^2}=dt$, we get

$I=\int e^t\left[t^2+2 t\right] dt \Rightarrow I=\int\left(t^2 e^t+2 e^t t\right) dt$

By integration by parts, we get

$$I=t^2 e^t-\int 2 t e^t dt-\int 2 t e^t dt+C$$

$$I=t^2 e^t+C=e^{\operatorname{tin}^{-1}(x)}\left(\tan^{-1} x\right)^2+C$$

Question105

$$\int \frac{dx}{(x-3)^{\frac{4}{5}}(x+1)^{\frac{6}{5}}}=$$

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Options:

A. $\frac{5}{4}\sqrt[5]{\frac{x-3}{x+1}}+C$

B. $\frac{5}{4}\left(\frac{x+1}{x-3}\right)^{\frac{1}{5}}+C$

C. $\frac{1}{5}\left(\frac{x-3}{x+1}\right)^{\frac{1}{5}}+c$

D. $\frac{5}{4}\left(\frac{x-3}{x+4}\right)^{\frac{4}{5}}+C$

Answer: A

Solution:

$$\text{Let } I=\int \frac{dx}{(x+1)^{\frac{6}{5}}(x-3)^{\frac{4}{5}}}$$
$$I=\int \frac{dx}{(x+1)^2\left(\frac{x-3}{x+1}\right)^{\frac{4}{5}}}$$

On putting $\frac{(x-3)}{(x+1)}=t$, then

$$dt=\frac{4}{(x+1)^2}dx$$
$$I=\int \frac{dt}{4t^{\frac{4}{5}}}=\frac{5}{4}t^{\frac{1}{5}}+C=\frac{5}{4}\left(\frac{x-3}{x+1}\right)^{\frac{1}{5}}+C$$

Question106

If $I_n=\int(\cos^n x+\sin^n x)dx$ and $I_n-\frac{n-1}{n}I_{n-2}=\frac{\sin x\cos x}{n}f(x)$, then $f(x)=$

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Options:

A. $\cos^{n-2}x+\sin^{n-2}x$

B. $\cos^{n-2}x-\sin^{n-2}x$

C. $\frac{\cos^{n-2}x-\sin^{n-2}x}{n}$

D. $\frac{\cos^{n-2}x+\sin^{n-2}x}{n}$

Answer: B

Solution:

Here, $I_n = \int (\cos^n x + \sin^n x)dx$

$I_n = \int \cos^{n-1} x \cos x dx + \int \sin^{n-1} x \sin x dx$

Using by parts,

$$I_n = \cos^{n-1} x \int \cos x dx - \int \left(\frac{d}{dx} (\cos^{n-1} x) \int \cos x dx \right) dx$$
$$+ \sin^{n-1} x \int \sin x dx - \int \left(\frac{d}{dx} (\sin^{n-1}) \int \left(\int \sin dx \right) dx \right.$$
$$\Rightarrow I_n = \cos^{n-1} \sin x + \int (n-1) \cos^{n-2} x \sin^2 x dx$$
$$- \sin^{n-1} x \cos x + \int (n-1) \sin^{n-2} x \cos^2 x dx$$
$$\Rightarrow I_n = \cos^{n-1} x \sin x - \sin^{n-1} x \cos x$$
$$+ \int (n-1) \cos^{n-2} x \left(1 - \cos^2 x\right) dx$$
$$+ \int (n-1) \sin^{n-2} x \left(1 - \sin^2 x\right) dx$$

$$\Rightarrow I_n = \cos^{n-1} x \sin x - \sin^{n-1} x \cos x$$
$$+ \int (n-1) \cos^{n-2} x dx$$
$$- \int (n-1) \cos^n x dx + \int (n-1) \sin^{n-2} x dx$$
$$- \int (n-1) \sin^n x dx$$

$$\Rightarrow I_n = \cos^{n-1} x \sin x - \sin^{n-1} x \cos x$$
$$+ \int (n-1) \cos^{n-2} x dx + \int (n-1) \sin^{n-2} x dx$$
$$- [(n-1) \int (\cos^n x + \sin^n x) dx$$

$$\Rightarrow I_n = \cos^{n-1} x \sin x - \sin^{n-1} \quad x \cos x$$
$$+(n-1)I_{n-2} - (n-1)I_n$$

$$\Rightarrow I_n(1 + n - 1) - (n-1)I_{n-2}$$
$$= \cos^{n-1} x \sin x - \sin^{n-1} x \cos x$$

$$\Rightarrow n\,(l_n) - (n-1)I_{n-2} = \cos^{n-1} x \sin x - \sin^{n-1} x \cos x$$
$$\Rightarrow I_n - \left(\frac{n-1}{n}\right) I_{n-2} = \frac{\sin x \cos x}{n} (\cos^{n-2} x - \sin^{n-2} x)$$
$$= \frac{\sin x \cos x}{n} f(x)$$
$$\therefore f(x) = \cos^{n-2} x - \sin^{n-2} x$$

Question107

If $f(x) = \int x^2 \cos^2 x \left(2x \tan^2 x - 2x - 6 \tan x\right) dx$ and $f(0) = \pi$, then $f(x) =$

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Options:

A. $x^2 \sin x + \pi$

B. $\cos x + \pi - 1$

C. $-x^3 \sin 2x + \pi$

D. $x^3 \cos 2x + \pi \cos x$

Answer: C

Solution:

$$\begin{aligned}
& f(x)=\int x^2 \cos ^2 x\left(2 x \tan ^2 x-2 x-6 \tan x\right) d x \\
& =\int\left(2 x^3 \sin ^2 x-2 x^3 \cos ^2 x\right) d x-3 \int x^2(2 \sin x \cdot \cos x) d x \\
& =-\int 2 x^3 \cdot \cos 2 x d x-3 \int x^2 \sin 2 x d x \\
& =-\left[2 x^3 \int \cos 2 x d x-\int 2 \cdot 3 x^2 \frac{\sin 2 x}{2} d x\right]-3 \int x^2 \sin 2 x d x \\
& =-x^3 \sin 2 x+3 \int x^2 \sin 2 x d x-3 \int x^2 \sin 2 x d x \\
& \quad f(x)=-x^3 \sin 2 x+C \\
& \therefore \quad f(0)=\pi \Rightarrow C=\pi \\
& \therefore \quad f(x)=-x^3 \sin 2 x+\pi
\end{aligned}$$

Question108

If $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}(x+\sqrt{x}) d x=e^{\sqrt{x}}[A x+B \sqrt{x}+C]+K$ **then** $A+B+C=$

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Options:

A. -2

B. 2

C. 4

D. -4

Answer: B

Solution:

$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}(x+\sqrt{x}) d x$

$= \int \left(\sqrt{x} e^{\sqrt{x}} + e^{\sqrt{x}}\right) dx = \int \sqrt{x} e^{\sqrt{x}} dx + \int e^{\sqrt{x}} dx$

Now, $\int \sqrt{x} \cdot e^{\sqrt{x}} dx$

Let $x = p^2$
$\Rightarrow dx = 2pdp$

$$= \int \sqrt{x} e^{\sqrt{x}} dx = \int p e^p (2pdp) = 2 \int p^2 e^p dp$$
$$= 2\left[p^2 e^p - 2 \int p e^p dp\right]$$
$$= 2\left[p^2 e^p - 2\left\{p e^p - e^p\right\}\right] + K$$
$$= 2\left[p^2 p^p - 2p e^p + 2e^p\right] + K$$
$$= 2\left[x e^{\sqrt{x}} - 2\sqrt{x} e^{\sqrt{x}} + 2e^{\sqrt{x}}\right] + K$$

$$\int e^{\sqrt{x}} dx = 2 \int p e^p dp = 2\left[p e^p - e^p\right] = 2\left[\sqrt{x} e^{\sqrt{x}} - e^{\sqrt{x}}\right]$$
$$\therefore \int \frac{e^{\sqrt{x}}}{\sqrt{x}}(x + \sqrt{x}) dx$$
$$= 2x e^{\sqrt{x}} - 4\sqrt{x} e^{\sqrt{x}} + 4e^{\sqrt{x}} + 2\sqrt{x} e^{\sqrt{x}} - 2e^{\sqrt{x}} + K$$
$$= e^{\sqrt{x}}(2x - 2\sqrt{x} + 2) + K$$
$$\therefore A = 2B = -2, C = 2$$
$$\therefore A + B + C = 2$$

Question109

If $\int \frac{1+\sqrt{\tan x}}{\sin 2x} dx = A \log \tan x + B \tan x + C$, then $4A - 2B =$

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Options:

A. -1

B. 2

C. 1

D. -2

Answer: A

Solution:

$\int \frac{1+\sqrt{\tan x}}{\sin 2x} dx$

$$= \int \operatorname{cosec} 2x dx + \int \frac{\sqrt{\tan x}}{\sin 2x} dx \Rightarrow \int \frac{1+\sqrt{\tan x}}{\sin x} dx$$

$$= \int \operatorname{cosec} 2x dx + \frac{\sqrt{\tan x}}{\sin 2x} dx$$

$$= \frac{1}{2} \log |\tan x| + \frac{1}{2} \int \frac{\sqrt{\sin x}}{\sqrt{\cos x} \sin x \cos x} dx$$

$$= \frac{1}{2} \log |\tan x| + \frac{1}{2} \int \frac{dx}{\sqrt{\cos x}\sqrt{\sin x}} \cos x \frac{\sqrt{\cos x}}{\sqrt{\cos x}}$$

$$= \frac{1}{2} \log |\tan x| + \frac{1}{2} \int \frac{\sec^2 x dx}{\sqrt{\tan x}}$$

$$= \frac{1}{2} \log |\tan x| + \frac{1}{2}(2) \cdot \sqrt{\tan x} + C$$

$$= \frac{1}{2} \log |\tan x| + \sqrt{\tan x} + C$$

$$\therefore 4A - 2B = 4 \times \frac{1}{2} - 2(l) = 2 - 2 = 0$$

Question110

$$\int \frac{1+\tan x \tan(x+a)}{\tan x \tan(x+a)} dx =$$

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Options:

A. $\tan a(\log(\sec(x+a)) + \log \sec x + C$

B. $\cot a(\log |\sin x| - \log |\sin(x+a)|) + C$

C. $\tan a \left(\log \left(\frac{\cos x}{\sin(x+a)}\right)\right) + C$

D. $\cot a \left(\log \frac{\sin(x+a)}{\cos(x+a)}\right) + C$

Answer: B

Solution:

$$\int \frac{1+\tan x \cdot \tan(x+a)}{\tan x \cdot \tan(x+a)} dx$$

$$= \int \frac{\cos(x)\cos(x+a) + \sin x \sin(x+a)}{\sin x \sin(x+a)} dx$$

$$= \int \frac{\cos(a)}{\sin x \cdot \sin(x+a)} dx = \frac{\cos a}{\sin a} \int \frac{\sin(x+a-x)}{\sin x \cdot \sin(x+a)} dx$$

$$= \cot a \int \frac{\sin(x+a)\cos x - \cos(x+a)\sin x}{\sin x \sin(x+a)} dx$$

$$= \cot a \left[\int \cot x dx - \int \cot(x+a) dx\right] + C$$

$$= \cot a[\log |\sin x| - \log |\sin(x+a)|] + C$$

Question111

Assertion (A) If $I_n = \int \cot^n x dx$, then$I_6 + I_4 = \frac{-\cot^5 x}{5}$

Reason (R) $\int \cot^n x dx = \frac{-\cot^{n-1} x}{n} - \int \cot^{n-2} x dx$

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Options:

A. A is false, R is false

B. A is true, R is true

C. A is true, R is false

D. A is false, R is true

Answer: C

Solution:

$$
\begin{aligned}
& I_n=\int \cot ^n x d x \\
& I_6+I_4=\int \cot ^6 x d x+\int \cot ^4 x d x \\
& =\int \cot ^4 x\left(\cot ^2 x+1\right) d x \\
& =-\int \cot ^4 x \cdot\left(-\operatorname{cosec}^2 x\right) d x \\
& =-\int(\cot x)^4 \cdot\left(-\operatorname{cosec}^2 x\right) d x=-\frac{(\cot x)^5}{5}
\end{aligned}
$$

$\therefore$ Assertion is true.

$$
\begin{aligned}
\int \cot ^n \quad x d x & =\int \cot ^{n-2} x \cdot \cot ^2 x d x \\
& =\int \cot ^{n-2} x \cdot\left(\operatorname{cosec}^2 x-1\right) d x \\
& =\int \cot ^{n-2} x \cdot \operatorname{cosec}^2 x \cdot d x-\int \cot ^{n-2} x d x \\
& =-\int(\cot x)^{n-2} x \cdot\left(-\operatorname{cosec}^2 x\right) d x-\int \cot ^{n-2} x d x \\
& =-\frac{(\cot x)^{n-1}}{n-1}-\int \cot ^{n-2} x d x
\end{aligned}
$$

$\therefore$ Reason is false.

Question112

If $I_n = \int \tan^n x dx$, and $I_0 + I_1 + 2I_2 + 2I_3 + 2I_4 + I_5 + I_6 = \sum\limits_{k=1}^{n} \frac{\tan^k x}{k}$, then $n =$

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Options:

A. 6

B. 5

C. 4

D. 3

Answer: B

Solution:

$$I_n = \int \tan^n x dx$$

$$I_0 + I_1 + 2I_2 + 2I_3 + 2I_4 + I_5 + I_6$$

$$= \int \tan^0 x dx + \int \tan^1 x dx + 2\int \tan^2 x dx + 2\int \tan^3 x dx$$

$$+2\int \tan^4 x dx + \int \tan^5 x \cdot dx + \int \tan^6 x dx$$

$$= \int \left[1 + \tan x + 2\tan^2 x + 2\tan^3 x + 2\tan^4 x + \tan^5 x + \tan^6 x\right] dx$$

$$= \int \left[1 + \tan^2 x + \left(\tan x + \tan^3 x\right) + \left(\tan^2 x + \tan^4 x\right) + \left(\tan^3 x + \tan^5 x\right) + \left(\tan^4 x + \tan^6 x\right)\right] dx$$

$$= \int \left[1 + \tan^2 x + \tan x\left(1 + \tan^2 x\right) + \tan^2 x\left(1 + \tan^2 x\right) + \tan^3 x\left(1 + \tan^2 x\right) + \tan^4 x\left(1 + \tan^2 x\right)\right] dx$$

$$= \int \left[\sec^2 x + \tan x \sec^2 x + \tan^2 x \cdot \sec^2 x + \tan^3 x \sec^2 x + \tan^4 x \sec^2 x\right] dx$$

$$= \int \left[1 + \tan x + \tan^2 x + \tan^3 x + \tan^4 x\right] \sec^2 x dx$$

$$= \int \sec^2 x \cdot dx + \int \left(\tan x + \tan^2 x + \tan^3 x + \tan^4 x\right) \sec^2 x dx$$

$$= \frac{\tan x}{1} + \frac{\tan^2 x}{2} + \frac{\tan^3 x}{3} + \frac{\tan^4 x}{4} + \frac{\tan^5 x}{5}$$

$$= \sum_{k=1}^{5} \frac{\tan^k x}{k} = \sum_{k=1}^{n} \frac{\tan^k x}{k} \Rightarrow n = 5$$

Question113

$$\int \frac{e^{\cot x}}{\sin^2 x}(2\log \operatorname{cosec} x + \sin 2x) dx =$$

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Options:

A. $-2 e^{\cot x} \log \left(\operatorname{cosec}^2 x\right)+C$

B. $-2 e^{\cot x} \log (\operatorname{cosec} x)+C$

C. $-2 e^{\cot x} \log (\operatorname{cosec} x+\sin x)+C$

D. $-2 e^{\cot x} \log (\operatorname{cosec} x-\cot x)+C$

Answer: B

Solution:

$$\text{Let } I=\int \frac{e^{\cos x}}{\sin ^2 x}(2 \log \operatorname{cosec} x+\sin 2 x) d x$$
$$I=\int e^{\cos x}\left(\log \operatorname{cosec}^2 x+\sin 2 x\right) \cdot \operatorname{cosec}^2 x d x$$

Let $\cot x=t \Rightarrow-\operatorname{cosec}^2 x d x=d t$

$$I=-\int e^t\left[\log \left(1+t^2\right)+\frac{2 t}{1+t^2}\right] d t$$
$$\left[\text{Here, } \frac{2 t}{1+t^2}=\frac{2 \cot x}{1+\cot ^2 x}=\frac{2 \cot x}{\operatorname{cosec}^2 x}\right.$$
$$\left.=\frac{2 \cos x / \sin x}{1 / \sin ^2 x}=\sin 2 x\right]$$
$$\Rightarrow \quad I=-e^t \cdot \log \left(1+t^2\right)+C$$
$$\left[\int e^x\left(f(x)+f^{\prime}(x)\right) d x=e^x \cdot f(x)+C\right]$$
$$\Rightarrow \quad I=-e^{\cot x} \log \operatorname{cosec}^2 x+C$$
$$\Rightarrow \quad I=-2 e^{\cot x} \log \operatorname{cosec} x+C$$

Question114

The parametric form of a curve is $x=\frac{t^3}{t^2-1} y=\frac{t}{t^2-1}$, then $\int \frac{d x}{x-3 y}=$

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Options:

A. $\frac{1}{2} \log \left(t^2-1\right)+C$

B. $2 \log \left(t\left(t^2-1\right)\right)+C$

C. $\frac{1}{4}\log\left(\frac{t}{t^2-3}\right)+C$

D. $\frac{5}{2}\log\left(t+\frac{1}{t^2}\right)+C$

Answer: A

Solution:

$$x=\frac{t^3}{t^2-1}, y=\frac{t}{t^2-1} \text{ and let } I=\int\frac{dx}{x-3y}$$

$$x=\frac{t^3}{t^2-1}$$

On differentiating w.r.t. t, we get

$$\frac{dx}{dt}=\frac{\left(t^2-1\right)\left(3t^2\right)-t^3(2t)}{\left(t^2-1\right)^2}$$

$$\Rightarrow \frac{dx}{dt}=\frac{3t^4-3t^2-2t^4}{\left(t^2-1\right)^2}$$

$$\Rightarrow dx=\frac{t^4-3t^2}{\left(t^2-1\right)^2}dt$$

$$\Rightarrow dx=\frac{t^2\left(t^2-3\right)}{\left(t^2-1\right)^2}dt$$

$$\Rightarrow I=\int\frac{1}{\frac{t^3}{t^2-1}-\frac{3t}{t^2-1}}\cdot\frac{t^2\left(t^2-3\right)}{\left(t^2-1\right)^2}dt$$

$$=\int\frac{\left(t^2-1\right)}{t\left(t^2-3\right)}\cdot\frac{t^2\left(t^2-3\right)}{\left(t^2-1\right)^2}dt$$

$$=\int\frac{t}{t^2-1}dt=\frac{1}{2}\int\frac{2t}{t^2-1}dt$$

$$=\frac{1}{2}\log\left(t^2-1\right)+C$$

Question115

If

$$\frac{2x^4-x^3+3x^2-x+4}{x^2-3x+2}=f(x)+\frac{A}{x-1}+\frac{B}{x-2}, \text{ then}$$

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Options:

A. $f(x)=2x^2+5x+14, A+B=39$

B. $f(x)=2x^2-5x+14, A+B=31$

C. $f(x)=2x^2+5x+14, A+B=31$

D. $f(x)=2x^2+5x+14, A=4, B=35$

Answer: C

Solution:

$\frac{2x^4-x^3+3x^2-x+4}{x^2-3x+2}$

$\because$ Degree of numerator is greater than the degree of denominator.

$\therefore$ First we divide,

$$
\begin{array}{r|l}
 & 2x^2+5x+14 \\
x^2-3x+2 & 2x^4-x^3+3x^2-x+4 \\
 & 2x^4-6x^3+4x^2 \\
 & -\quad+\quad- \\
\hline
 & 5x^3-x^2-x+4 \\
 & 5x^3-15x^2+10x \\
 & -\quad+\quad- \\
\hline
 & 14x^2-11x+4 \\
 & 14x^2-42x+28 \\
 & -\quad+\quad- \\
\hline
 & 31x-24
\end{array}
$$

$$
\begin{aligned}
\text{or } & =\frac{2x^4-x^3+3x^2-x+4}{x^2-3x+2} \\
& =\left(2x^2+5x+14\right)+\frac{31x-24}{(x-2)(x-1)} \quad \text{.....(i)}
\end{aligned}
$$

Let $\frac{31x-24}{(x-2)(x-1)}=\frac{A}{x-1}+\frac{B}{x-2}$

$\Rightarrow 31-24=-A(x-2)+B(x-1)$ ..... (ii)

Put $x=1$

$\Rightarrow 31-24=-A \Rightarrow A=-7$

Put $x=2$ in Eq. (ii)

$31\times 2-24=0+B \Rightarrow B=38$

$\therefore$ From Eq. (i) $f(x)=2x^2+5x+14$

and $A+B=-7+38=31$

Question116

If $f'(x)=x+\frac{1}{x}$, then $f(x)$ is equal to

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Options:

A. $x^2 + \log(x) + c$

B. $\frac{x^2}{2} + \log(x) + c$

C. $x + \log(x) + c$

D. $\frac{x}{2} + \log(x) + c$

Answer: B

Solution:

$f'(x) = x + \frac{1}{x}$, then integrate it

$f(x) = \int (x + \frac{1}{x}) dx + c = \frac{x^2}{2} + \log x + c$

Question117

If $f(x) = \frac{1}{(\cos^2 x)\sqrt{1+\tan x}}$, then its antiderivative $F(x) =$ ...., given, $F(0) = 4$

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Options:

A. $\sqrt{1 + \tan x} + 4$

B. $\frac{2}{3}(1 + \tan x)^{\frac{3}{2}}$

C. $2(\sqrt{1 + \tan x} + 1)$

D. $\sqrt{1 + \tan x} + 2$

Answer: C

Solution:

$$f(x) = \frac{1}{\cos^2 x\sqrt{1 + \tan x}} = \frac{\sec^2 x}{\sqrt{1 + \tan x}}$$
$$\Rightarrow \quad \int f(x) dx = \int \frac{\sec^2 x}{\sqrt{1 + \tan x}} dx$$
$$\Rightarrow \text{ Let } 1 + \tan x = u, \text{ then } \sec^2 x dx = du$$
$$\Rightarrow \quad \int f(x) dx = \int \frac{du}{\sqrt{u}} = 2\sqrt{u} + C$$
$$\Rightarrow \quad F(x) = 2\sqrt{1 + \tan x} + C$$
$$\text{Given, } F(0) = 4 = 2\sqrt{1} + C \Rightarrow C = 2$$
$$\therefore \quad F(x) = 2(\sqrt{1 + \tan x} + 1)$$

Question118

If the primitive of $\cos(\log x)$ is $f(x)\{\cos(g(x))+\sin(h(x))\}$, then which among the following is true?

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Options:

A. $h'(x)=\frac{-1}{x}$

B. $f'(x)=\frac{1}{2}$

C. $g'(x)=\log(x)$

D. $h(x)=\frac{x}{2}$

Answer: B

Solution:

Given,

$$\int \cos(\log x)dx = f(x)\{\cos(g(x))+\sin(h(x))\}$$

$$I=\int_{\text{II}} \underset{\text{I}}{\cos(\log x)}dx$$

{ using by-part method of integration }

$$=\cos(\log x)\cdot x+\int \frac{\sin(\log x)}{x}\cdot xdx$$

$$=\cos(\log x)\cdot x+\int \sin(\log x)dx$$

$$=\cos(\log x)\cdot x+\left[\sin(\log x)\cdot x-\int \frac{\cos(\log x)}{x}\cdot xdx\right]$$

$$I=x\cdot\cos(\log x)+x\cdot\sin(\log x)-I$$

$$2I=x(\cos(\log x)+\sin(\log x))$$

$$\Rightarrow I=\frac{x}{2}(\cos(\log x)+\sin(\log x))$$

$$\therefore\quad f(x)=\frac{x}{2}\Rightarrow f'(x)=\frac{1}{2}$$

Question119

$\int \frac{\sec x}{\sqrt{\sin(2x+\theta)+\sin\theta}}dx$ **is equal to**

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Options:

A. $\sqrt{(\tan x+\tan \theta) \sec \theta}+c$

B. $\sqrt{2(\tan x+\tan \theta) \sec \theta}+c$

C. $\sqrt{2(\sin x+\tan \theta) \sec \theta}+c$

D. $\sqrt{2(\cos x+\tan \theta) \sec \theta}+c$

Answer: B

Solution:

$$
\begin{aligned}
& \int \frac{\sec x}{\sqrt{\sin (2 x+\theta)+\sin \theta}} d x=I \text { (say) } \\
& I=\int \frac{\sec x}{\sqrt{\sin 2 x \cos \theta+\cos 2 x \sin \theta+\sin \theta}} d x \\
& I=\int \frac{\sec x}{\sqrt{\sin 2 x \cos \theta+(\cos 2 x+1) \sin \theta}} d x \\
& =\int \frac{\sec x}{\sqrt{2 \cos x(\sin x \cos \theta+\cos x \sin \theta)}} d x \quad\left[\because \cos 2 x+1=2 \cos ^2 x\right] \\
& =\int \frac{\sec x}{\sqrt{2 \cos ^2 x \cos 2 x \theta(\tan x+\tan \theta)}} d x
\end{aligned}
$$

$$
\begin{aligned}
& =\frac{1}{\sqrt{2 \cos \theta}} \int \frac{\sec ^2 x}{\sqrt{\tan x+\tan \theta}} d x \\
& \text { Let } \tan x+\tan \theta=u \Rightarrow \sec ^2 x d x=d u \\
& =\frac{1}{\sqrt{2 \cos \theta}} \int \frac{d u}{\sqrt{u}}=\frac{1}{\sqrt{2 \cos \theta}} \cdot 2 \cdot \sqrt{u}+c \\
& =\frac{\sqrt{2}}{\sqrt{\cos \theta}} \cdot \sqrt{\tan x+\tan \theta}+c=\sqrt{\frac{2(\tan x+\tan \theta)}{\cos \theta}}+c \\
& =\sqrt{2(\tan x+\tan \theta) \sec \theta}+c
\end{aligned}
$$

Question120

Given, $\frac{3x-2}{(x+1)^2(x+3)}=\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{x+3}$, then $4A+2B+4C$ is equal to

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Options:

A. 5

B. -5

C. -3

D. 3

Answer: B

Solution:

$$\frac{3x-2}{(x+1)^2(x+3)}=\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{(x+3)}$$
$$\Rightarrow 3x-2=A(x+1)(x+3)+B(x+3)+C(x+1)^2$$
Putting $x=-1$
$$-5=2B$$
$$\Rightarrow \quad B=\frac{-5}{2}$$
Putting $x=-3$
$$-11=4C\Rightarrow C=\frac{-11}{4}$$

Putting $x=-3$

$-11=4C\Rightarrow C=\frac{-11}{4}$

Putting $x=0$

$$-2=3A+3B+C\Rightarrow -2=3A-\frac{15}{2}-\frac{11}{4}$$
$$\Rightarrow \quad \frac{33}{4}=3A\Rightarrow A=\frac{11}{4}$$
$$\Rightarrow 4A+2B+4C=11-5-11=-5$$

Question121

$\int \frac{\sin\alpha}{\sqrt{1+\cos\alpha}}d\alpha$ is equal to

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Options:

A. $-2\sqrt{2}\cos\left(\frac{\alpha}{2}\right)+C$

B. $2\sqrt{2}\cos\left(\frac{\alpha}{2}\right)+C$

C. $\sqrt{2}\cos\left(\frac{\alpha}{2}\right)+C$

D. $-\sqrt{2}\cos\left(\frac{\alpha}{2}\right)+C$

Answer: A

Solution:

$$\int \frac{\sin \alpha d\alpha}{\sqrt{1+\cos\alpha}} = \int \frac{\sin\alpha\sqrt{1-\cos\alpha}}{\sqrt{1-\cos^2\alpha}} d\alpha$$
$$= \int \sqrt{1-\cos\alpha} d\alpha$$
$$= \sqrt{2}\int \sin\frac{\alpha}{2} d\alpha$$
$$= -2\sqrt{2}\cos\frac{\alpha}{2}$$

Question122

If $\int \frac{\cos 4x+1}{\cot x-\tan x} = k\cos 4x + C$, then k is equal to

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Options:

A. $\frac{-1}{2}$

B. $\frac{-1}{8}$

C. $\frac{-1}{3}$

D. $\frac{-1}{5}$

Answer: B

Solution:

$$\int \frac{1+\cos 4x}{\cot x-\tan x} dx = \int \frac{2\cos^2 2x}{\frac{\cos 2x}{\cos x \sin x}} dx$$
$$= \int (2\cos 2x \cos x \sin x) = \frac{1}{2}\int 2\cos 2x \sin 2x dx$$
$$= \frac{1}{2}\int \sin 4x dx = \frac{-1}{8}\cos 4x + C \Rightarrow k = -\frac{1}{8}$$

Question123

If $\int \left[\cos(x)\cdot\frac{d}{dx}(\operatorname{cosec}(x)\right] dx = f(x) + g(x) + c$ then $f(x)\cdot g(x)$ is equal to

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Options:

A. $x\cot(x)$

B. $x \tan(x)$

C. $x \cos(x)$

D. 1

Answer: A

Solution:

$$\int -\cos x \operatorname{cosec} x \cot x d x$$
$$=-\int \cot^2 x d x$$
$$=\int\left(1-\operatorname{cosec}^2 x\right) d x=x+\cot x+C$$
$$\therefore \quad f(x)=x \text{ and } g(x)=\cot x$$
$$f(x) \cdot g(x)=x \cot x$$

Question124

If $\int \frac{(2x+1)^6}{(3x+2)^8} dx = P\left(\frac{2x+1}{3x+2}\right)^Q + R$, then $\frac{P}{Q}$ is equal to

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Options:

A. $\frac{1}{7^2}$

B. $\frac{1}{7}$

C. 7^2

D. 7

Answer: A

Solution:

$$\int \frac{(2x+1)^6}{(3x+2)^8} dx$$
$$\int \left(\frac{2x+1}{3x+2}\right)^6 \left(\frac{1}{3x+2}\right)^2 dx$$
Let $\frac{2x+1}{3x+2} = t$
$$\Rightarrow \quad \frac{(3x+2)2-(2x+1)3}{(3x+2)^2} = \frac{dt}{dx} \Rightarrow \left(\frac{1}{3x+2}\right)^2 dx = dt$$
$$\Rightarrow \quad \int t^6 dt = \frac{t^7}{7} + C \Rightarrow P = \frac{1}{7}, Q = 7$$
$P/Q = 1/49 = 1/72$

Question125

Which of the following is partial fraction of $\frac{-x^2+6x+13}{(3x+5)(x^2+4x+4)}$ is equal to

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Options:

A. $\frac{3}{3x+5} + \frac{-1}{x+2} + \frac{2}{(x+2)^2}$

B. $\frac{2}{3x+5} + \frac{-1}{x+2} + \frac{3}{(x+2)^2}$

C. $\frac{-1}{3x+5} + \frac{2}{x+2} + \frac{3}{(x+2)^2}$

D. $\frac{3}{3x+5} + \frac{2}{x+2} + \frac{-1}{(x+2)^2}$

Answer: B

Solution:

$$\frac{-x^2+6x+13}{(3x+5)(x^2+4x+4)} = \frac{-x^2+6x+13}{(3x+5)(x+2)^2}$$
$$\Rightarrow \quad \frac{-x^2+6x+13}{(3x+5)(x+2)^2} = \frac{A}{3x+5} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$
$$\Rightarrow \quad -x^2+6x+13 = A(x+2)^2 + B(3x+5)(x+2) + C(3x+5) \ldots \text{(i)}$$

It is an identity

$\therefore$ True for every value of x.

Put $x = -2$ in Eq. (i),

$$-(-2)^2 + 6(-2) + 13 = A \cdot 0 + B \cdot 0 + C(3(-2)+5)$$
$$\Rightarrow -4 - 12 + 13 = -C \Rightarrow C = 3$$

Put $x = \frac{-5}{3}$ in Eq. (i), we get

$$-\left(\frac{-5}{3}\right)^2+6\left(\frac{-5}{3}\right)+13$$

$$=A\left(\frac{-5}{3}+2\right)^2+B\cdot 0+C\cdot 0$$

$$\Rightarrow \quad \frac{-25}{9}-10+13=A\cdot\frac{1}{9}$$

$$\Rightarrow \quad \frac{-25}{9}+3=\frac{A}{9}\Rightarrow A=2$$

Put $x=0$ in Eq. (i), we get

$$13=4A+10B+5C$$
$$\Rightarrow 13=8+10B+15$$
$$\Rightarrow 10B=13-23 \text{ or } B=-1$$
$$\therefore \frac{-x^2+6x+13}{(3x+5)(x^2+4x+4)}$$
$$=\frac{2}{3x+5}+\frac{-1}{x+2}+\frac{3}{(x+2)^2}$$

Question126

$\int \frac{1+x+\sqrt{x+x^2}}{\sqrt{x}+\sqrt{1+x}}dx$ **is equal to**

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Options:

A. $\frac{1}{2}(\sqrt{1+x})+c$

B. $\sqrt{1+x}+c$

C. $2(1+x)^{3/2}+c$

D. $\frac{2}{3}(1+x)^{3/2}+c$

Answer: D

Solution:

$$\text{Let } I=\int\frac{(1+x)+\sqrt{x+x^2}}{\sqrt{x}+\sqrt{1+x}}dx$$
$$=\int\frac{(1+x)+\sqrt{x}\sqrt{1+x}}{\sqrt{x}+\sqrt{1+x}}dx$$
$$=\int\frac{\sqrt{1+x}(\sqrt{1+x}+\sqrt{x})}{(\sqrt{1+x}+\sqrt{x})}dx$$
$$=\int\sqrt{1+x}dx$$

Let $1+x=t^2\Rightarrow dx=2tdt$

$\therefore I=2\int t^2dt=2\frac{t^3}{3}+C=\frac{2}{3}(1+x)^{3/2}+c$

Question127

$\int(\cos x) \log \cot \left(\frac{x}{2}\right) dx$ **is equal to**

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Options:

A. $(\sin x) \log \cot \left(\frac{x}{2}\right)+c$

B. $(\cos x) \log \cot \left(\frac{x}{2}\right)+c$

C. $(\sin x) \log \cot \left(\frac{x}{2}\right)+x+c$

D. $(\sin x) \log \cot \left(\frac{x}{2}\right)-x+c$

Answer: C

Solution:

$$\text{Let } I=\int \underset{\text{II}}{(\cos x)} \underset{\text{I}}{\log \cot (x/2)} dx$$

$$=\log \cot \frac{x}{2} \cdot \int \cos x dx$$

$$-\int\left[\frac{d}{dx}\left(\log \cot \frac{x}{2}\right) \int \cos x dx\right] dx \quad \text{[integrating by parts]}$$

$$=\log \left(\cot \frac{x}{2}\right) \cdot \sin x-\int \frac{1}{\cot \frac{x}{2}}$$

$$\cdot-\operatorname{cosec} \frac{x}{2} \cdot \frac{1}{2} \cdot \sin x dx$$

$$=(\sin x) \log \cot \frac{x}{2}+\int 1 dx$$

$$=(\sin x) \log \cot \frac{x}{2}+x+c$$

Question128

$\int \sqrt{e^{4x}+e^{2x}} dx$ **is equal to**

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Options:

A. $\frac{1}{2} e^x\left(\sqrt{e^{2x}+1}\right)+\frac{1}{2} \sinh^{-1}\left(e^x\right)+c$

B. $\frac{1}{2} e^x\left(\sqrt{e^{2 x}+1}\right)+\sinh ^{-1}\left(e^x\right)+c$

C. $\frac{1}{2}\left(\sqrt{e^{2 x}+1}\right)+\frac{1}{2} \sinh ^{-1}\left(e^x\right)+c$

D. $\sqrt{e^{4 x}+e^{2 x}}+\sqrt{e^{2 x}+1}+c$

Answer: A

Solution:

$$
\begin{aligned}
& I=\int \sqrt{e^{4 x}+e^{2 x}} d x=\int e^x \sqrt{e^{2 x}+1} d x \\
& \text{Put } e^x=t \Rightarrow e^x d x=d t \\
& \therefore \quad I=\int \sqrt{t^2+1} d t \\
& =\frac{1}{2} t \sqrt{t^2+1}+\frac{1}{2.1} \sinh ^{-1}(t)+c \\
& =\frac{1}{2} e^x \sqrt{e^{2 x}+1}+\frac{1}{2} \sinh ^{-1}\left(e^x\right)+c
\end{aligned}
$$

Question129

If $\int \frac{1}{1+\sin x} d x=\tan (f(x))+c$, then $f^{\prime}(0)$ is equal to

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Options:

A. 0

B. 1

C. $\frac{1}{2}$

D. $\frac{-1}{2}$

Answer: C

Solution:

$$
\begin{aligned}
& \int \frac{1}{1+\sin x} d x=\tan (f(x))+c \\
& \Rightarrow \int \frac{1-\sin x}{\cos ^2 x} d x=\tan (f(x))+c
\end{aligned}
$$

$\Rightarrow \quad \int\left(\sec^2 x-\sec x \tan x\right) d x=\tan (f(x))+c$
$\Rightarrow \quad \tan x-\sec x+C=\tan (f(x))+c$
$\Rightarrow \quad \tan \left(\frac{\pi}{4}+\frac{x}{2}\right)=\tan (f(x))$
$\Rightarrow \quad -\tan \left(\frac{-\pi}{4}-\frac{x}{2}\right)=\tan (f(x))$
$\Rightarrow \quad f(x)=-\frac{\pi}{4}+\frac{x}{2} \Rightarrow f^{\prime}(x)=\frac{1}{2}$
At $\quad x=0, f^{\prime}(0)=\frac{1}{2}$

Question130

$\int \frac{e^x(x+3)}{(x+5)^3} d x$ **is equal to**

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Options:

A. $\frac{e^x}{(x+5)^2}+C$

B. $e^x(x+5)^2+C$

C. $e^x(x+3)^2+C$

D. $\frac{e^x}{(x+3)^2}+C$

Answer: A

Solution:

$$\text{Let } I=\int \frac{e^x(x+3)}{(x+5)^3} d x=\int e^x\left[\frac{1}{(x+5)^2}-\frac{2}{(x+5)^3}\right] d x$$
$$\int\left[f(x)+f^{\prime}(x)\right] e^x d x=e^x f(x)+C$$
$$\Rightarrow \quad I=\frac{e^x}{(x+5)^2}+C$$

Question131

If $\int \frac{(x-1)^2}{\left(x^2+1\right)^2} d x=\tan ^{-1}(x)+g(x)+k$**, then** $g(x)$ **is equal to**

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Options:

A. $\tan^{-1}\left(\frac{x}{2}\right)$

B. $\frac{1}{x^2+1}$

C. $\frac{1}{2(x^2+1)}$

D. $\frac{2}{x^2+1}$

Answer: B

Solution:

$$\int \frac{(x-1)^2}{(x^2+1)^2} dx = \int \left(\frac{1}{x^2+1} - \frac{2x}{(x^2+1)^2}\right) dx$$
$$= \tan^{-1}(x) - \int \frac{2x}{(x^2+1)^2} dx$$

Let $x^2+1=t \Rightarrow 2xdx = dt$

$$= \tan^{-1}(x) - \int t^{-2} dt = \tan^{-1}(x) + \frac{1}{t} + k$$
$$\Rightarrow \int \frac{(x-1)^2}{(x+1)^2} dx = \tan^{-1}(x) + \frac{1}{x^2+1} + k$$

Comparing with $\int \frac{(x-1)^2}{(x^2+1)^2} dx = \tan^{-1}(x) + g(x) + k$ we have $g(x) = \frac{1}{x^2+1}$

Question132

If $\int \frac{1-(\cot x)^{2021}}{\tan x+(\cot x)^{2022}} dx = \frac{1}{A} \log\left|(\sin x)^{2023} + (\cos x)^{2023}\right| + c$, then A is equal to

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Options:

A. 2020

B. 2021

C. 2022

D. 2023

Answer: D

Solution:

$$\frac{1}{A}\log\left|(\sin x)^{2023}+(\cos x)^{2023}\right|+C$$

$$=\int\frac{1-(\cot x)^{2021}}{\tan x+(\cot x)^{2022}}dx=\int\frac{1-\frac{(\cos x)^{2021}}{(\sin x)^{2021}}}{\frac{\sin x}{\cos x}+\frac{(\cos x)^{2022}}{(\sin x)^{2022}}}dx$$

$$=\int\frac{(\sin x)^{2021}-(\cos x)^{2021}}{(\sin x)^{2023}+(\cos x)^{2023}}\cdot\cos x\sin x dx$$

$$=\int\frac{\cos x(\sin x)^{2022}-\sin x(\cos x)^{2022}}{(\sin x)^{2023}+(\cos x)^{2023}}dx$$

$$\because\int\frac{f'(x)}{f(x)}dx=\log|f(x)|+C$$

$$\Rightarrow\quad\frac{1}{A}\log\left|(\sin x)^{2023}+(\cos x)^{2023}\right|+C$$

$$=\frac{1}{2023}\log\left|(\sin x)^{2023}+(\cos x)^{2023}\right|+C$$

$$\Rightarrow\quad A=2023$$
